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ON SIGNALLING AND SCREENING

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L'esprit pionnier

On Signalling and Screening

Anastasios Dosis*

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Abstract

The two games usually employed to model markets with asymmetries of information are the signalling game and the screening game. In the signalling game, an equilibrium may not be efficient due to the arbitrariness in the off-the-equilibrium-path beliefs. In the screening game, a pure strategy Nash equilibrium may fail to exist because of “cream-skimming” deviations. Perhaps surprisingly, I show how in a game that combines signalling and screening, an equilibrium generically exists and is efficient.

KEYWORDS: Signalling, screening, information economics, existence, efficiency

JEL CLASSIFICATION: D82, D86

1 INTRODUCTION

■ **Motivation.** Almost every modern microeconomics textbook features a full chapter on information economics. Within this chapter, one finds two distinctive games used to describe markets with asymmetries of information. On the one hand, there is the signalling game, introduced by Spence (1973) and further extended by Maskin and Tirole (1992), in which an informed party proposes a menu of contracts to an uninformed party who accepts or rejects. On the other hand, there is the screening game, originally introduced by Rothschild and Stiglitz (1976), in which at least two symmetric, uninformed parties compete in menus of contracts to attract an informed party. One cannot help but notice that the equilibrium sets of these two games vary drastically. In the signalling game, a generic indeterminacy of inefficient equilibria can be sustained due to the arbitrariness

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in the off-the-equilibrium-path beliefs. In the screening game, a pure strategy equilibrium may fail to exist because the uninformed parties may have “cream-skimming” profitable deviations. A natural question arises: What happens in a game that embeds both signalling and screening? Perhaps surprisingly, I show that in such a game an equilibrium generically exists and is efficient.

To demonstrate this result, I employ a stylised competitive insurance market. In this market, there is a consumer who can suffer a costly accident and seeks for insurance. Two symmetric firms provide insurance. There are two possible types for the consumers: high-risk and low-risk. The high-risk type is more prone to suffering the accident than the low-risk type. An insurance contract specifies a premium and an indemnity in case the accident occurs. In the first part of the paper, I contrast the sets of equilibria of the signalling game *vis-à-vis* the screening game. In the signalling game, the consumer proposes a menu of contracts to both companies. If at least one company accepts the proposal, the consumer selects a contract from at most one of the companies who accepted. If both companies reject, then the consumer remains uninsured. In the screening game, the two companies simultaneously and independently compete in menus of contracts. The consumer then selects a contract from at most one company. The least-costly-separating allocation is defined as the allocation that maximises the payoff of both types of the consumer within the set of incentive compatible allocations that are individually rational for the company type-by-type.

As shown in Maskin and Tirole (1992), every incentive compatible allocation that makes positive profits (relative to the prior beliefs) and weakly dominates the least-costly-separating allocation can be supported as a perfect Bayesian equilibrium in the signalling game. On the contrary, as shown in Mas Colell et al. (1995) and Jehle and Reny (2011), a pure strategy Nash equilibrium in the screening game exists and is unique, in terms of the payoffs, if and only if the least-costly-separating allocation is efficient. Therefore, the two games agree in the predictions they make about the outcome of the market, if and only if the least-costly-separating allocation is efficient.

I then analyse a simple combination of the two games, which I call the *signalling-screening* game. Similar to the signalling game, in the first stage, the consumer proposes a menu of contracts. In the second stage, each company accepts or rejects. If both companies reject the game ends. If at least one company accepts, the game moves to the third stage. Similar to the screening game, in the third stage, each company who accepted can propose a menu of contracts to the consumer. In the fourth stage, the consumer selects at most one company and a contract from those he proposed in the first stage, or those that this company proposed in the third stage. The result is, perhaps, striking. If the least-costly-separating allocation is efficient, then the equilibrium set of the signalling-screening game is payoff-equivalent to both the signalling and the screening games. If the least-costly-separating allocation is not efficient, then an allocation is an equilibrium allocation if and only if it is efficient, and weakly dominates the least-costly-separating allocation.

To give an intuitive explanation of this result note that the first two stages of the game are the signalling stages. As in the signalling game, with his offer, every type can guarantee that his payoff does not fall below the least-costly-separating allocation. The third stage is the screening stage. This stage is indispensable to avoid the inefficiency result of

the signalling game. Suppose that the consumer has made a “non-efficient” offer in the first stage and is unable to deviate, as in the signalling game, because both companies would infer that the deviation comes from the high-risk and, therefore, reject any dominating offer. The addition of the third stage of the game solves this problem. In particular, even if the consumer is unable to deviate, I show that one of the companies always has an incentive to do so. Therefore, a necessary condition for an equilibrium allocation is that this is efficient. A natural question then arises: Why does a company not deviate from an efficient menu of contracts, as in the screening game? This is where the fourth stage of the game plays a role. In particular, note that in the signalling-screening game, the consumer has the right to either select one of the menus proposed by one of the companies or his own menu by any company who accepted his offer. This very fact creates “barriers to deviations”.¹ Indeed, one can construct a sequentially rational strategy for the consumer such that both types propose an efficient allocation in the first stage and both types select to contract with the company that offers a “cream-skimming” menu of contracts in the third stage if it does so. Recall that no type is worse off, compared to his offer by contracting with any company because, by definition, he has access to his offer in the fourth stage. This suffices to render any deviation on the side of the companies unprofitable.

□ **Related Literature.** Since the seminal contribution of Spence (1973), a vast literature unfolded in signalling games. Overall, signalling games have been very successful in explaining a variety of observed phenomena, e.g. corporate financial structure (Myers and Majluf (1984)), limit pricing (Milgrom and Roberts (1982)), advertising (Milgrom and Roberts 1986), etc., that conventional economic theory faced difficulties to explain. The indeterminacy of equilibria, inevitably, led researchers to search for appropriate refinements that restrict the off-the-equilibrium path beliefs, e.g. Cho and Kreps (1987), Banks and Sobel (1987). One of the various applications of signalling games and the closest to this paper is to the theory of contracts. As I already mentioned, Maskin and Tirole (1992) examine a signalling game in which the informed party is the principal who makes a take-it-or-leave-it offer of a mechanism to a uninformed party. I extensively analyse the equilibria of this signalling game in the simplest possible model and compare these to the signalling-screening.

Following the contribution of Rothschild and Stiglitz (1976), screening games have been used in the modelling of competition in markets with adverse selection. The striking result of the non-existence of equilibrium established by Rothschild and Stiglitz (1976), inevitably attracted considerable attention and motivated researchers to search for alternative (weaker) definitions of equilibrium. Wilson (1977) and Riley (1979) are the most well-known definitions. Applications of screening games are found in the analysis of insurance, credit and labour markets, in which uninformed companies try to “screen” informed consumers through menus of contracts, e.g. Miyazaki (1977), Spence (1978), Bester (1985) etc. Recently, researchers have shown a renewed interest in analysing games in which the set of equilibrium allocations includes only efficient allocations, e.g. Asheim and Nilssen (1996), Diasakos and Koufopoulos (2011), Mimra and Wambach (2011), Netzer and Scheuer (2014), Picard (2014) and Dosis (2016a). In Dosis (2016a), I take a mech-

¹The name “barriers to deviations” was suggested by a referee.

anism design approach. I construct a decentralised mechanism in which an equilibrium always exists and is efficient. The mechanism takes the following form: All companies in the market simultaneously and independently offer two menus of contracts. I call one of these menus the “private menu” and the other the “public menu”. The distinctive characteristic of the mechanism is that the union of all public menus needs to be offered by all active companies. The private menu concerns only the company who offers it. In Dosis (2016), I show that the set of equilibrium allocations includes the entire set of constrained Pareto efficient allocations. On the contrary, in this paper, I characterise the equilibrium set of a game that combines signalling and screening. The set of equilibrium allocations of the signalling-screening game differs from this in Dosis (2016a) because the payoff of no type can fall below the least-costly-separating allocation. To the best of my knowledge, this is the first paper that combines signalling and screening.

The remainder of the paper is organised as follows: In Section 2, I present the stylised insurance market. In Section 3, I describe the signalling and the screening games and their sets of equilibria. In Section 4, I introduce the signalling-screening game and I analyse its set of equilibria. In Section 5, I provide a brief conclusion and discuss possible extensions.

2 THE MODEL

■ **The Insurance Market.** There is a single consumer and two insurance companies. The consumer can be one of two possible types $i = H, L$. Both companies believe that the probability of the consumer to be of type i is λ_i . Each type i starts with wealth W and can suffer a damage equal to d with probability θ_i , where $\theta_H > \theta_L$. A contract specifies a premium p and a benefit (indemnity) b . A contract is denoted as $c = (p, b) \in \mathbb{R}_+^2$. The expected utility of type i from contract c is $U_i(c) = (1 - \theta_i)u(W - p) + \theta_i u(W - d - p + b)$, where $u' > 0$ and $u'' < 0$. The expected profit of the company who provides contract c is equal to $\pi_i(c) = p - \theta_i b$.

□ **Allocations.** An *allocation* is a pair of contracts indexed by the set of types, i.e. $(c_H, c_L) \in \mathbb{R}_+^4$. In the jargon of mechanism design, an allocation is simply a direct revelation mechanism. An allocation (c_H, c_L) is *incentive compatible* if and only if $U_H(c_H) \geq U_H(c_L)$ and $U_L(c_L) \geq U_L(c_H)$. One of the benchmark incentive compatible allocations is what is called the *least-costly-separating* allocation.

Definition 2.1. *The least-costly-separating allocation is denoted as (\hat{c}_H, \hat{c}_L) and satisfies the following conditions: $\hat{b}_H = d$, $\hat{p}_H = \theta_H d$, and $U_L(\hat{c}_L) = U_L(\hat{c}_H)$, $\pi_L(\hat{c}_L) = 0$.*

The least-costly-separating allocation is depicted in Figure 1. The horizontal axis represents the benefit, the vertical axis represents the premium. The two straight lines are the zero profit lines, one for each type, and the curves are the indifference curves of the two types.

I now define efficient allocations.

Definition 2.2. *An allocation (c_H, c_L) is efficient if and only if: (i) $U_H(c_H) \geq U_H(c_L)$ and $U_L(c_L) \geq U_L(c_H)$, (ii) $\sum_{i=H,L} \lambda_i \pi_i(c_i) \geq 0$, and, (iii) there exists no allocation $(\tilde{c}_H, \tilde{c}_L)$ that satisfies (i) and (ii) and moreover $U_i(\tilde{c}_i) \geq U_i(c_i)$ for every i with the inequality being strict for at least one i .*

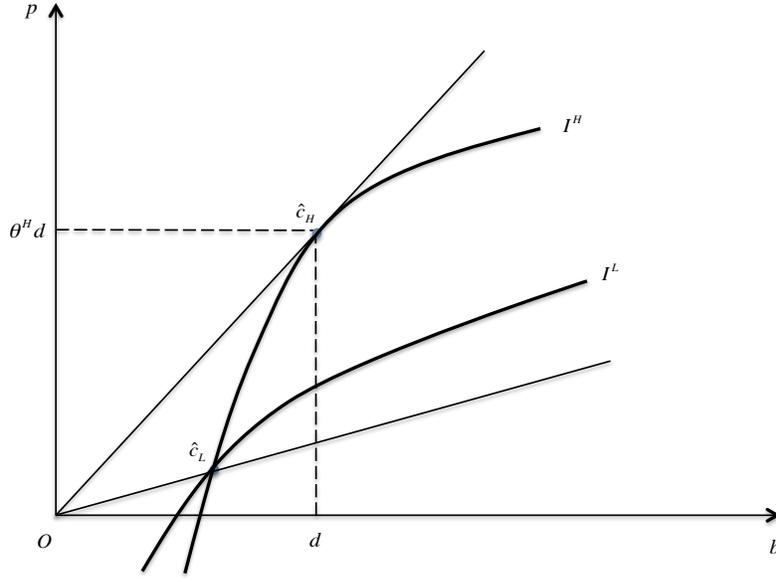


Figure 1: The least-costly-separating allocation

Note that efficiency is defined with respect to the payoff of the consumer. In other words, I concentrate on the subset of Pareto efficient allocations that maximise the payoff of the consumer, i.e. a weighted average of the utilities of the two types, subject to the incentive compatibility constraint and the individual rationality constraint of a representative company relative to the prior beliefs. Under the stated assumptions, one can prove the following result:

Lemma 2.3. *If (c_H, c_L) is an efficient allocation, then $\sum_{i=H,L} \lambda_i \pi_i(c_i) = 0$.*

Proof. Consider an incentive compatible allocation (c_H, c_L) such that $\sum_{i=H,L} \lambda_i \pi_i(c_i) > 0$. Take allocation $(\tilde{c}_H, \tilde{c}_L)$ where $u(W - \tilde{p}_i) = qu(W - p_i) + (1 - q)u(W - \theta_L d)$ and $u(W - d - \tilde{p}_i + \tilde{b}_i) = qu(W - d - p_i + b_i) + (1 - q)u(W - \theta_L d)$. Clearly, $U_i(\tilde{c}_i) > U_i(c_i)$ for every $i = H, L$ and $0 < q < 1$. Because $u'' < 0$, $W - \tilde{p}_i < q(W - p_i) + (1 - q)(W - \theta_L d)$, or

$$\tilde{p}_i > qp_i - (1 - q)\theta_L d \quad (1i)$$

Similarly,

$$\tilde{p}_i - \tilde{b}_i > q(p_i - b_i) - (1 - q)(\theta_L d) \quad (2i)$$

Multiplying (1i) by $1 - \theta_i$ and (2i) by θ_i and summing up, one obtains

$$\tilde{p}_i - \theta_i \tilde{b}_i > q(p_i - \theta_i b_i) - (1 - q)\theta_L d \quad (3i)$$

which can be re-written as

$$\pi_i(\tilde{c}_i) > q\pi_i(c_i) - (1 - q)\theta_L d \quad (4i)$$

Multiplying (4i) by λ_i and summing up over $i = H, L$, one obtains

$$\sum_i \lambda_i \pi_i(\tilde{c}_i) > q \sum_i \lambda_i \pi_i(c_i) - (1 - q)\theta_L d \quad (5)$$

which for q small enough is strictly higher than zero. This means that (c_H, c_L) cannot be efficient \square

An *equilibrium allocation* is an allocation that results as the equilibrium outcome of a game between the consumer and the companies when all players play their equilibrium strategies.

3 SIGNALLING VS SCREENING

The first benchmark model related to this paper is a variant of the signalling model introduced by Maskin and Tirole (1992). Maskin and Tirole (1992) study an extensive form game with three stages. I henceforth refer to this game as the *signalling game*. In the first stage, the consumer proposes a pair of contracts. In the second stage, each company accepts or rejects the proposal. If both companies reject, the game ends. If at least one company accepts, the game moves to the third stage. In the third stage, the consumer selects a company and a contract from those he offered in Stage 1.² Maskin and Tirole (1992) prove that *an allocation (c_H, c_L) is an equilibrium allocation in the signalling game, if and only if it satisfies the following conditions:*

$$(IC_{H,L}) \quad U_H(c_H) \geq U_H(c_L)$$

$$(IC_{L,H}) \quad U_L(c_L) \geq U_L(c_H)$$

$$(LCS_i) \quad U_i(c_i) \geq U_i(\hat{c}_i) \quad \forall i$$

$$(PP) \quad \sum_{i=H,L} \lambda_i \pi_i(c_i) \geq 0$$

The formal proof can be found in Maskin and Tirole (1992). The intuition behind the characterisation of the equilibrium is the following. $(IC_{H,L})$, $(IC_{L,H})$ and (PP) in the “only if” part is rather straightforward. An equilibrium allocation needs to be incentive compatible and individually rational for the companies. For (LCS_H) and (LCS_L) , consider any of the types and assume that his equilibrium payoff is strictly lower than this provided by the least-costly-separating contract. Suppose that the consumer proposes in the first stage a strictly incentive compatible menu of contracts that is arbitrarily close to the least-costly-separating menu of contracts. Maskin and Tirole (1992) show that under the underlying assumptions such a menu exists. Then, at least, one company should accept the proposal because regardless of her belief, this menu makes strictly positive profits which contradict the definition of equilibrium.³ Because this applies to every contract arbitrarily close to the least-costly-separating menu of contracts, the lower bound in

²The main difference between the signalling game explained in textbooks and this I examine here is the possibility given to the consumer to propose a pair of contracts instead of a single contract. This difference notwithstanding, the game is a standard signalling game.

³This relies on the fact that the menu of contracts proposed by the consumer is strictly incentive compatible. Therefore revealing his true type is a strictly dominant strategy in the third stage for the consumer.

the equilibrium payoff of every type is the payoff from the least-costly-separating contract. Now suppose that both types propose in the first stage a menu of contracts satisfying $(IC_{H,L}), (IC_{L,H}), (LCS_H), (LCS_L)$ and (PP) . Such a menu is depicted in Figure 2.⁴ One company accepts and in the third stage both types select contract \bar{c} . The question is whether, for all possible other menus of contracts that are strictly preferred by at least one type, there exist beliefs such that both companies reject the proposal. Maskin and Tirole (1992) show that this is so.⁵

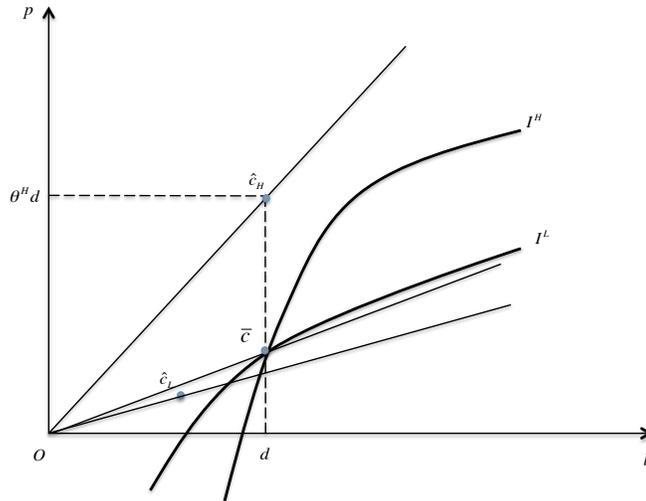


Figure 2: (\bar{c}, \bar{c}) is an equilibrium menu of contracts in the signalling game.

There are two fundamental implications of Proposition 1. First, the equilibrium of the signalling game is unique if and only if the least-costly-separating allocation is efficient. Second, if the least-costly-separating menu of contracts is not efficient, then the set of equilibrium menus of contracts includes menus that are not efficient. This result is a consequence of the arbitrariness of the off-the-equilibrium path beliefs and it is common in games that include some information transmission.

The second benchmark model is this examined by Rothschild and Stiglitz (1976). Here, I present a variant of this model as it was presented in textbooks such as Mas Colell et al. (1995) and Jehle and Reny (2011).⁶ These authors consider the following game, which henceforth I will call the *screening game*. There are two stages. In the first stage, each com-

⁴The menu of contracts depicted in Figure (2) corresponds to the only “pooling” menu of contracts that is efficient. On that see Crocker and Snow (1985).

⁵Note in Figure 2 that for every incentive compatible menu of contracts that dominates the least-costly-separating menu of contracts, one of the two contracts is not individually rational for the company. The off-the-equilibrium-path beliefs then can be determined as follows. For every menu of contracts that provides higher payoff to at least one of the types than menu (\bar{c}, \bar{c}) , both companies believe that the consumer is of type L with certainty. Based on these beliefs, both companies should reject the proposal of the consumer if he was to make an offer that provides to at least one type a payoff higher than menu (\bar{c}, \bar{c}) . Based on these strategies, no type has an incentive to deviate unilaterally.

⁶The reason for this choice is that the models examined in Mas Colell et al. (1995) and Jehle and Reny (2011) have more “modern” game-theoretic flavour and allow companies to offer menus of contracts. See also Dosis (2016) for the generalisation of their results.

pany simultaneously and independently offers a menu of contracts. In the second stage, the consumer selects a contract from at most one company. Rothschild and Stiglitz (1976) show that *if the least-costly-separating allocation is efficient, then it is the unique equilibrium allocation in the screening game. If the least-costly-separating allocation is not efficient, then an equilibrium in pure strategies in the screening game does not exist.*

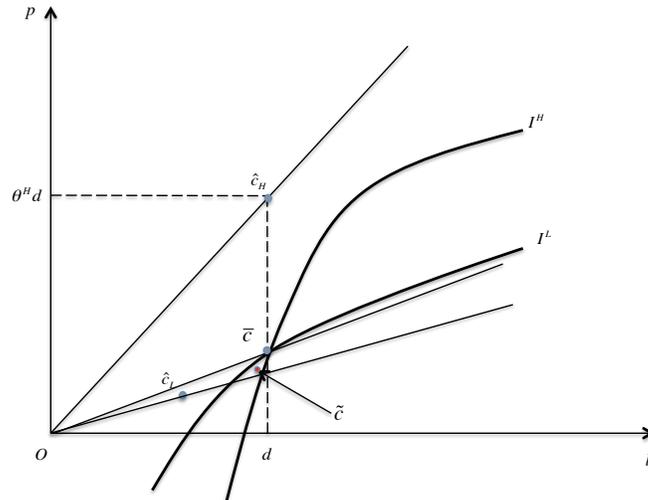


Figure 3: (\bar{c}, \bar{c}) is not an equilibrium in the screening game because (\tilde{c}, \tilde{c}) constitutes a “cream-skimming” deviation. If Company A offers menu of contracts (\bar{c}, \bar{c}) , then Company B, by offering menu (\tilde{c}, \tilde{c}) , attracts only the low-risk type and makes strictly positive profits.

Therefore, the set of pure strategy equilibria is a singleton (in terms of payoffs) when the least-costly-separating menu of contracts is efficient but it is the empty set when the least-costly-separating menu of contracts is not efficient. For formal proofs one can refer to Mas Colell et al. (1995) and Jehle and Reny (2011) or Dosis (2016b). The intuition behind the result is the following. First, one can show that a menu of contracts is an equilibrium menu of contracts only if it is efficient. This is the “Bertrand-type” feature of the game. Indeed, for any non-efficient menu of contracts, there exists another menu that if introduced by the some company results in higher profits. Second, when the least-costly-separating allocation is efficient, an equilibrium can be constructed in which both companies offer this menu of contracts and no company can unilaterally, profitably deviate. When the least-costly-separating allocation is not efficient, then efficiency entails cross-subsidisation. In other words, one type necessarily subsidises the other type as this is depicted in Figure (2). One can then show that there exists no equilibrium in which the contract for at least one type makes strictly positive profits. For every such menu, there always exists a profitable deviation as this is depicted in Figure (3). There is an apparent contradiction with the first point and hence, an equilibrium in pure strategies does not exist.

4 THE SIGNALLING-SCREENING GAME

■ **The Signalling-Screening Game.** Now let us consider a model that combines both games. As such consider the following four-stage game, which I call the *signalling-screening* game. In the first stage, the consumer proposes a menu of contracts (c_1, c_2) . In the second stage, each company accepts or rejects. If both companies reject, then the game ends. If at least one company accepts, the game moves to the third stage. In the third stage, each company who accepted can propose a new menu of contracts. In the fourth stage, the consumer selects one company and either one of the contracts he proposed in the first stage, or a contract proposed by that the company.

A *strategy* for the consumer specifies for every possible type a menu of contracts in the first stage of the game and a choice of a contract for every possible history of play in the fourth stage of the game. A strategy for a company specifies for every possible history of play, a decision to accept or reject in the second stage and for every possible history a choice of a menu of contracts in the third stage of the game. A *belief system* specifies a probability distribution over the two types for every possible history of play, i.e. for every menu of contracts in the first stage.

I will only consider the pure-strategy perfect Bayesian equilibria of this game. A *perfect Bayesian equilibrium* is a set of strategies, one for each player, and a belief system, such that: (i) the strategy of each player is sequentially rational given the beliefs, and, (ii) the beliefs are determined by Bayes rule given the players equilibrium strategies.

A, perhaps, striking result is obtained:

Proposition 4.1. *An allocation (c_H, c_L) is an equilibrium allocation if and only if: (i) it is efficient, and, (ii) $U_i(c_i) \geq U_i(\hat{c}_i)$ for every i .*

The idea of the proof can be readily summarised as follows. First, consider the “only if” part. To begin with, note that the proof that $u_i(c_i) \geq u_i(\hat{c}_i)$ for every i is identical to Maskin and Tirole (1992) and hence, for evident reasons, is omitted. The following lemma facilitates the proof.

Lemma 4.2. *In every pure strategy equilibrium, both types offer the same menu of contracts.*

PROOF: Suppose that there exists a pure strategy equilibrium in which the two types propose a different menu of contracts. It is straightforward to see that in any pure strategy equilibrium, the maximum payoff for the high-risk type is $u_L(\hat{c}_L)$. In a different occasion, after the proposal of the high-risk type, both companies would reject. Now suppose that in equilibrium, after his proposal, the low-type contracts with Company A. Because Company B can always make an offer in the third stage of the game, the equilibrium contract of the low-risk type is $(\theta_L d, \theta_L d)$, i.e. the contract that the low-type would receive in case the types were observable. This, however, contradicts the no unilateral deviation condition of the equilibrium because in such a case the high-type would have an incentive to deviate and pretend to be low-type. Q.E.D.

Assume now that there exists an equilibrium allocation (c_H, c_L) that is not efficient. This means, due to Lemma (2.3), that there exists an allocation (\bar{c}_H, \bar{c}_L) such that $u_i(\bar{c}_i) >$

$u_i(c_i)$ for every i and $\sum_i \lambda_i \pi_i(\bar{c}_i) > 0$. I showed in Lemma (4.2) that, in every pure-strategy equilibrium, both types offer the same menu of contracts. There are two possible cases. Either $\pi_i(c_i) = 0$ for every i , or, $\pi_j(c_j) < 0$ for some j . In the first case, it is possible that in equilibrium the two types contract with a different company. In the second case, in equilibrium both types necessarily contract with the same company. It is easy to see that in both cases, one of the companies (or both) has a profitable unilateral deviation. In particular, by offering the pair of contracts (\bar{c}_H, \bar{c}_L) it attracts both types and makes strictly positive profits. This contradicts the no-unilateral deviation condition in the definition of equilibrium and hence (c_H, c_L) cannot be an equilibrium allocation.

For the “if” part, consider the following strategies : Both types offer the pair of contracts (\bar{c}_H, \bar{c}_L) , i.e. an efficient allocation satisfying $u_i(\bar{c}_i) \geq u_i(\hat{c}_i)$ for every i , and both contract with Company A if both companies offer (\bar{c}_H, \bar{c}_L) . If some company, unilaterally, offers a different pair of contracts $(\tilde{c}_H, \tilde{c}_L)$ then both types contract with this company. Each company accepts and offers (\bar{c}_H, \bar{c}_L) if the offer of the consumer is (\bar{c}_H, \bar{c}_L) . Each company rejects if the offer of the consumer $(\tilde{c}_1, \tilde{c}_2) \neq (\bar{c}_H, \bar{c}_L)$ is such that $\max_{c \in \{\tilde{c}_1, \tilde{c}_2\}} u_H(c) > u_H(\hat{c}_H)$. If both types offer (\bar{c}_H, \bar{c}_L) , then the posterior beliefs remain equal to the prior beliefs. For every other offer, the posterior beliefs are updated to $(0, 1)$, i.e. both companies believe with certainty that the type made the deviation is high-risk.⁷ One can easily certify that the strategies described above constitute equilibrium strategies. Based on the beliefs held by the two companies, no player has an incentive to unilaterally deviate.

□ **Remarks.** A point that is a key for the existence of equilibrium, and hence deserves further discussion, is that in the signalling-screening game, the consumer has the right to either select one of the menus proposed by one of the companies or his menu by any company who accepted his offer. A first question that comes to mind is how the equilibrium set varies when the companies can entirely disregard the offer of the consumer and just offer their menus of contracts. In such a case, one can easily show that the equilibrium set of the game becomes payoff equivalent to that of the screening game. But then the stage where the consumer makes an offer becomes questionable. In particular, why is the consumer allowed to offer something that he can never select? For a signalling-screening game to make sense, it must allow the informed party to have access to all possible offers.

A related question is whether this choice can become, somehow, endogenous. To that extent, let me highlight that a payoff equivalent game is one in which the consumer selects in the first stage of the game a menu of contracts and whether he commits to this menu. Commitment means that any company who accepts his offer needs to let the consumer select a contract out of the menu he offered in the first stage even if this company makes another offer in the third stage. In this game, one can show that if the least-costly-separating menu of contracts is efficient, commitment plays no particular role. Nonetheless, if the least-costly-separating menu of contracts is not efficient, then in equilibrium the consumer offers an efficient menu of contracts and commits to this menu. Then no company has a unilateral profitable deviation.

⁷Recall that the same characterisation of beliefs was given in the characterisation of the equilibrium set in Maskin and Tirole (1992).

The last comment is the following: One can think the addition of the screening part after the signalling part as a “refinement” of the equilibrium set of the signalling game.⁸ In particular, note that the signalling game admits a large number of equilibria. One can argue that most of them are implausible because they are supported by “unreasonable” off-the-equilibrium path beliefs. Equilibria exist which are strictly dominated for both types. As we saw, the addition of the screening game eliminates all these equilibria and hence the addition of the screening part “refines” the set of equilibrium allocations of the signalling game.

5 CONCLUSION

In this paper, I revisited the two common games used to model markets with asymmetries of information: the *signalling game* and the *screening game*. I showed that in a game that embeds both signalling and screening, the equilibrium set is not empty, unlike in the pure *screening game*, and every equilibrium allocation is efficient, unlike in the *signalling game*.

Two assumptions certainly deserve more discussion. The first is the restriction to two types. None of the results rely on this assumption. In particular, one can straightforwardly realise that the argument given for the proof of the “if” part in Proposition (4.1) straightforwardly extends to environments with any finite number of types. The “only if” part of Proposition 2 is slightly more subtle. Nonetheless, by invoking the well-known *inscrutability principle* of Myerson (1983), one can show that there is no loss of generality to restrict ourselves to equilibria in which no information revelation takes place, i.e. all types offer the same menu of contracts in the first stage, substituting that way (4.2). By doing so, the argument I provided for the “only if” part seems to hold with any finite number of types. Assuming only two types simplifies the exposition and allows for diagrammatic analysis.

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