Interest Rates and Investment Under Competitive Screening and Moral Hazard

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The pioneering spirit

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Abstract

This paper studies the effect of (market) interest rate changes on investment under competitive screening and moral hazard. Lower (higher) rates ease (hinder) the provision of incentives to entrepreneurs with positive NPV projects to invest in their best project but hinder (ease) banks' efforts to distinguish them from entrepreneurs with negative NPV projects. This might result in a hump-shaped investment curve. Under low rates, screening through limit pricing leaves insufficient profits to low-wealth entrepreneurs to invest in their best project, and consequently, several project qualities might co-exist in equilibrium. Several testable and other implications on the effectiveness of unconventional monetary policy to boost investment are discussed.

KEYWORDS: Interest rates, entrepreneurial wealth, investment, competitive screening, moral hazard

JEL CLASSIFICATION: D82, E30, E44, E58, G01, G21

1 INTRODUCTION

The precise relationship between interest rates and investment has intrigued economists at least since Fisher (1930) and Keynes (1936). The neoclassical (e.g., Haavelmo (1960), Jorgenson (1963)) and Tobin's-q (i.e., Tobin (1969)) theories of investment predict that interest rates and investment are inversely related since interest rates affect the user cost (equivalently, the replacement cost) of capital and, consequently, the profitability of investing in capital assets. More recent theories based on financial market imperfections

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(e.g., Bernanke and Gertler (1995)) highlight additional potential channels that reinforce this inverse relationship.[1] It is argued there that changes in interest rates affect the balance sheets of credit-constrained firms and, consequently, their ability to access the financial market. However, the recent Japanese, US and Euro-area experiences tell a different story. In all these places, interest rates hit their zero lower bounds but had only a modest impact on business investment.[2] To account for a potential non-monotonic relationship between the market interest rate and investment, this paper provides an alternative theory that relies on asymmetric information problems in financial markets and the role that interest rates play in determining the ability of banks to screen entrepreneurs and providing incentives for investment in high-quality projects.

In particular, I study a model with wealth-heterogeneous, financially constrained entrepreneurs who seek to finance risky projects using banks. In the baseline model, there are two possible project qualities (i.e., entrepreneurs' types), which differ in their riskiness: high-type projects second-order stochastically dominate low-type projects. Moreover, only high-type projects exhibit positive net present value (NPV). Banks cannot identify the true quality of a project, which leads to a standard "lemons" problem. Adverse selection is pernicious and could lead to a complete market shutdown if entrepreneurs were totally cashless. Therefore, entrepreneurial wealth serves as a screening device and partially alleviates the lemons problem.

The role of the market interest rate is twofold: it is the rate at which (i) banks raise the necessary funds from depositors to grant loans and (ii) entrepreneurs can save excess funds. For an entrepreneur with a low-type project, investing a share of her wealth in the project entails a cost. Because banks wish to drive out of the market all low-type projects, they demand that borrowers invest as much of their wealth as possible in the project (i.e., have "skin in the game"). However, because the lower the wealth level, the more difficult it becomes to discourage an entrepreneur with a low-type project from borrowing, in addition and up to a threshold, banks demand as repayment a sufficiently high share of the project's return. This is socially costly, as it drives out of the credit market, alongside low-type projects, low-wealth entrepreneurs with high-type projects who would have borrowed and invested had information been perfect. As a consequence, a decrease in the market rate has an adverse effect: it decreases a low-type's implicit cost of investing in the risky project and, therefore, forces banks to raise their lending standards to prevent the former from borrowing. This drives out of the market a larger share of high-type projects and, hence, decreases aggregate investment.

In the second part of the paper, I extend the model to account for the possibility of an endogenous choice of projects. Entrepreneurs can now select among different projects, each of which entails a different cost of operation. A share of entrepreneurs has only negative NPV projects, whereas the remaining share has at least one strictly positive NPV project. Each entrepreneur's projects can be ranked according to first-order stochastic

[1]See Mishkin (1995, 2007) or Kashyap and Stein (1995) for the potential transmission channels of monetary policy and reviews of the literature.

[2]The following quote from the Economist describes this puzzle: "*IT'S ONE of the fundamental lessons of any introductory economics course: lower interest rates, when all else remains equal, leads [sic] to higher levels of investment. But today, after several years of near-zero interest rates and only modest increases in investment to show for it, some economists are claiming just the opposite...*" Free Exchange, The Economist, November 12, 2015.

dominance, with higher expected return projects entailing a higher cost of operation. Entrepreneurs continue to be privately informed, but now even the project choice is unobservable to banks. Hence, alongside adverse selection, a moral hazard problem can potentially arise.

Entrepreneurial wealth serves two important functions. First, as in the baseline model, it facilitates screening positive from negative NPV projects. Second, it provides a discipline device for entrepreneurs to adopt higher quality projects. The market interest rate plays a significant role in these two functions. When interest rates are low, screening is difficult, and hence, banks need to ration a significant share of entrepreneurs with positive NPV projects. In that region, the minimum wealth required by banks as skin in the game is decreasing in the interest rate. Moreover, because banks price loan contracts in a *limit pricing* manner, low-wealth entrepreneurs are left with insufficient rents to invest in higher quality projects. Therefore, in equilibrium, several project qualities might coexist. When interest rates are high, screening becomes costless but banks face difficulties in providing incentives to entrepreneurs to invest in higher quality projects. Therefore, the minimum wealth required as skin in the game is increasing in the interest rate. This means that investment may be hump-shaped in the interest rate.

The model offers several insights regarding the relationship between interest rates and investment when financial markets are imperfect. Perhaps the most profound is that competitive screening might attenuate the stimulative effects of monetary policy. Moreover, the model provides several other implications regarding the risk-taking channel of monetary policy and the effect of interest rate changes on the quality of investment. Finally, several testable implications are discussed.

- *Related Literature.* The paper builds on the seminal contributions of Stiglitz and Weiss (1981) (hereafter, SW) and Holmstrom and Tirole (1997) (hereafter, HT).

SW consider a credit market with a continuum of projects and adverse selection. The equilibrium is characterised by pooling at the same repayment rate as in Akerlof (1970). The main result is that debt financing can lead to credit rationing in equilibrium. This paper differs from SW in several respects. First, entrepreneurs are heterogenous in two dimensions: the riskiness of their projects and their entrepreneurial wealth. Second, unlike SW, banks use wealth to screen entrepreneurs.[3] Therefore, the model is closer to Rothschild and Stiglitz (1976) (hereafter, RS) and Bester (1985, 1987), although, unlike RS, a free-entry equilibrium always exists and is unique. Third, in this paper, there is an interplay between competitive screening and moral hazard that provides fruitful insights.

HT also emphasise the role of entrepreneurial wealth in mitigating agency problems. As in the present paper, entrepreneurs choose between two different projects that are ranked according to first-order stochastic dominance, with higher expected return projects entailing a higher cost of operation. As expected, with no initial wealth, there is no financing the because incentives to invest in high-quality projects are weak. Entrepreneurial wealth increases the cost of shirking, and hence, for a sufficiently high share in the project, an entrepreneur is able to guarantee financing. The main difference from HT is that in

[3] SW also consider a case in which entrepreneurs are wealth heterogeneous (i.e., Section III, p. 402). However, they do not allow banks to use wealth to screen out the different types, which is key in this paper.

this paper competitive screening and moral hazard co-exist. Therefore, the safe interest rate differently affects the margin of entrepreneurs who acquire funds in equilibrium. Moreover, unlike HT, in this paper, many project qualities may co-exist in equilibrium, as screening hinders the provision of incentives to entrepreneurs to invest in higher quality projects.

Other papers also highlight the importance for borrowers to have "skin in the game" as a mechanism to mitigate asymmetric information problems in financial markets. In Leland and Pyle (1977), a risk-averse investor of a high-quality asset can signal herself by (inefficiently) selling only a portion of the asset to risk-neutral investors. Related results are obtained in DeMarzo and Duffie (1999), Biais and Mariotti (2005), Chemla and Hennessy (2014) and Vanasco (2017), in which an originator wishing to optimally design a security to raise cash signals the quality of the asset through costly retention of future cash flows. Bester (1985, 1987) highlights the role of collateral as a possible sorting device in the SW model.[4] In my model, there is no collateral, as entrepreneurial wealth is highly liquid and, hence, non-pledgeable. However, entrepreneurial wealth plays an important role in providing incentives, similar to that played by collateral in Bester (1985, 1987). Moreover, the focus of the papers is different, as this paper mainly studies the effect of interest rate changes on investment.

This paper is also related to the literature that studies the effects of adverse selection in macroeconomics and finance (e.g., Myers and Majluf (1984), Kurlat (2013), Malherbe (2014), Bigio (2015)). These papers are closer in spirit to the original work of Akerlof (1970), in contrast to my model that involves screening as in Spence (1973) and Rothschild and Stiglitz (1976).

Closely related are the papers of Dell'Ariccia and Marquez (2006) and Nenov (2016). Dell'Ariccia and Marquez (2006) examine the effect of information on bank collateral requirements (i.e., lending standards) in a related model of competitive screening. There are several differences between my model and that of Dell'Ariccia and Marquez (2006). First, the focus of the present paper is on the effect of interest rate changes on investment, in contrast to Dell'Ariccia and Marquez (2006), who mainly focus on the effect of information on lending standards. Second, in the present paper, unlike Dell'Ariccia and Marquez (2006), entrepreneurs are wealth-heterogeneous, which introduces an extensive margin. In my model, even entrepreneurs with high-quality projects but insufficient wealth do not receive financing in equilibrium, unlike Dell'Ariccia and Marquez (2006), who find that all entrepreneurs with high-quality projects receive financing. Finally, another important difference is the modelling of competition in loan contracts by financial intermediaries. To avoid equilibrium existence problems, Dell'Ariccia and Marquez (2006) model competition as in Hellwig (1987), unlike this paper, where competition is modelled as in RS. Dell'Ariccia and Marquez (2006) find that pooling equilibria can be sustained, unlike the present paper, which finds that a unique fully separating equilibrium exists.

Nenov (2016) studies a model of a production economy with a capital asset (i.e., land)

[4] Related, although to a lesser extent, are the papers by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), who show how small shocks are amplified through a "financial accelerator" when there is a wedge between the implicit costs of internal and external financing. In Bernanke and Gertler (1989), lenders can enforce repayment only through costly monitoring, while in Kiyotaki and Moore (1997), monitoring is infinitely costly, and therefore, loan contracts need to be fully collateralised.

that entrepreneurs use to produce output. The price of land determines the entrepreneurs' outside option, and hence, in the least-costly separating equilibrium, demand for land is increasing in its price. Nenov (2016) then studies the effect of productivity shocks on production in this particular equilibrium. There are several differences between this paper and Nenov (2016). First, my model involves both adverse selection and moral hazard, unlike Nenov (2016), who focuses on adverse selection. This leads to qualitatively different results. Second, in my model, a unique separating equilibrium exists. In Nenov (2016), multiple separating and pooling equilibria might exist. This is mostly due to the difference in our modelling of the financial market. I model the financial market as a pure screening game (i.e., Rothschild and Stiglitz (1976)), whereas Nenov (2016) models this in a Walrasian manner (i.e., Gale (1992, 1996, 2001), Dubey and Geanakoplos (2002), Citanna and Siconolfi (2016), Azevedo and Gottlieb (2017)). Third, in my model, entrepreneurs own fixed-investment projects, and in equilibrium, even some entrepreneurs of high-quality projects are subject to credit rationing and hence do not invest (i.e., an extensive margin exists), unlike Nenov (2016), who finds that all entrepreneurs of high-quality projects invest in equilibrium.[5] Finally, Nenov (2016) assumes that, in addition to imperfect information, a problem of limited commitment in loan repayments exists. I do not impose such an assumption.

Finally, related in terms of scope is Chetty (2007), who argues that when firms make irreversible investments, an increase in the interest rate changes both the opportunity cost of capital and the cost of delaying investment to acquire information. This can give rise to an inverse U-shaped demand for investment.

The remainder of the paper is organised as follows. In Section 2, I describe the baseline model with exogenously given projects and characterise its unique equilibrium. In Section 3, I extend the model to account for an endogenous choice of projects and characterise its unique equilibrium. In Section 4, I discuss several policy and testable implications of the model. In Section 5, I conclude the paper. All formal proofs are in Appendix A.

2 The Model with Exogenously Given Projects

A. Entrepreneurs and Banks

- *Entrepreneurs.* There are two periods (henceforth, period one and period two) and one good that is used both for consumption and investment. All agents consume solely in period two. There is a continuum of entrepreneurs, and each has a new investment project that he wishes to undertake and a level of wealth. Entrepreneurs are heterogeneous in two respects: the quality of their project and their amount of wealth. The quality of the project can be high or low, $i \in \{H, L\}$, and is privately known by the entrepreneur, whilst wealth W belongs to $[0, +\infty)$.[6] For simplicity, suppose that i and W are independently

[5] Due to this extensive margin and the fact that banks price contracts in a *limit-pricing* manner, low-wealth entrepreneurs acquire only a partial share of the project's surplus. This effect is absent in Nenov (2016).

[6] The assumption of infinite wealth is clearly unrealistic and in fact not necessary for any of the results. It is only employed to simplify the characterisation of aggregate investment as a function of the interest rate and facilitate comparative statics. Alternatively, one could assume that the upper bound of wealth is

distributed, with λ_i denoting the (marginal) probability i to be realised and $F(W)$ the (marginal) cumulative distribution function (cdf) of W, which is assumed to be continuous with full support and differentiable with pdf $f(W) = F'(W)$. Suppose also that $\int_0^{+\infty} W dF(W) < +\infty$. By investing I in period one, a type-i entrepreneur can realise in period two a payoff equal to X_i with probability π_i or equal to zero with probability $1 - \pi_i$.[7] In other words, upon undertaking the project, the entrepreneur has a positive probability of going bankrupt in period two. For notational simplicity, let $\Delta X = X_L - X_H$ and $\overline{\pi} = \lambda_H \pi_H + \lambda_L \pi_L$.

The (net) risk-free real interest rate (or simply market rate) from period one to period two is $r \geq 0$. This rate refers to the opportunity cost of not investing a unit of the good in the financial market. I assume that this is exogenously given.

Henceforth, I impose the following restriction on parameters.

Assumption 1. *(i)* $\Delta X > 0$, *(ii)* $X_H > \max\{\frac{\pi_L X_L}{\pi_H}, \frac{I(1+r)}{\pi_H}\}$, *and, (iii)* $X_L < \frac{I(1+r)}{\overline{\pi}}$

Assumption 1 has several implications. First, (i) and (ii) imply that type H's project second-order stochastically dominates type L's project. Following SW, this specification is standard in the credit-rationing literature (e.g., Bester (1987), Hellwig (1987)).[8,9] Second, (ii) implies that type-H entrepreneurs possess positive NPV projects. Third, because $\overline{\pi} > \pi_L$, (iii) implies that type-L entrepreneurs possess negative NPV projects.[10] Moreover, adverse selection could have led to a collapse of the financial market had entrepreneurs been cashless. Therefore, as will become clear shortly, wealth can partially alleviate the information asymmetry problem.[11]

- *Banks and Loan Contracts.* There is a potentially infinite number of risk-neutral banks that accept deposits and offer loan contracts. Banks are symmetric, profit-maximising entities. I make three assumptions about the space of feasible loan contracts. First, entrepreneurs are protected by limited liability. Second, wealth can be invested in the project but cannot be pledged as collateral for future loan repayments. The second assumption presumes that any wealth not invested in the project can be consumed before loan repayments take

sufficiently high.

[7]For simplicity, I call an entrepreneur with a quality-i project a type-i entrepreneur.

[8]SW assume that a type-L project is a mean-preserving spread of type H's project. I assume that the expected return of a type-H project is strictly higher than the expected return of a type-L project.

[9]Second-order stochastic dominance represents the higher risk of a low-quality project relative to a high-quality project. Its main implication is that, under standard debt financing, when banks increase the debt-repayment rate, it is the relatively safer projects that are driven out of the market, which might result in a riskier loan portfolio and hence lower bank profits. This modelling strategy attempts to replicate the outcomes in Akerlof (1970)'s market for *lemons*. An alternative, equivalent modelling strategy is to assume that $X_H = X_L = X$ but entrepreneurs with higher probability to succeed have a strictly higher outside option than entrepreneurs with lower probability to succeed. This modelling strategy was employed in an earlier version of this paper and yields qualitatively identical results.

[10]Below, I analyse the effect of a change in the interest rate on aggregate investment. Assumption 1 restricts attention to a certain range of interest rates. Implicit in this assumption is that type-L entrepreneurs have a negative NPV project even when the market rate hits its lower bound (i.e., when the nominal interest rate hits its zero lower bound).

[11]Wealth can only "partially" alleviate the information asymmetry problem because, as I show below, not all type-H entrepreneurs can guarantee financing.

place, and therefore, no bank will accept it as collateral.[12] Third, for a type-L entrepreneur, the return ΔX over X_H is not observable or/and verifiable by a court of law.[13] Due to these three assumptions and the fact that upon failure of the project an entrepreneur goes bankrupt, there is no loss of generality in concentrating on simple, risky-debt contracts.[14] A risky-debt contract (or, for simplicity, contract) is denoted by $\psi = (S, R) \in \mathbb{R}_+^2$ and specifies the amount that an entrepreneur needs to invest S and the amount from the project's return, if this succeeds, that is pledged for repayment of the loan R.[15]

I assume that entrepreneurs do not renege on their repayment promises, thereby abstracting from commitment or imperfect enforcement issues, which allows me to concentrate on the effect of ex ante information asymmetries on aggregate investment. I further assume that wealth is observable by banks. This assumption can be justified if wealth refers to retained earnings or other financial assets that are reported in financial statements and can be easily verified by a bank's experts who examine loan applications. Furthermore, as will become clear, this assumption plays no role in the qualitative features of the results. Its main purpose is to simplify the notation and analysis, as it permits loan contracts to be made contingent on an entrepreneur's observable wealth.[16]

- *Timing of Events and Equilibrium.* Period one has three stages. In the first stage, banks enter the market, and each offers at most one contract. In the second stage, entrepreneurs each select at most one contract from at most one bank. In the last stage, those entrepreneurs who borrowed from a bank invest in the risky project, whereas those who did not invest receive the market rate. In period two, the project's return is realised (i.e., success or failure), and all agents consume. The timing of events is summarised in Figure 1.

A *menu of contracts* $\mu : [0, +\infty] \twoheadrightarrow [0, I] \times [0, X_H]$ is a correspondence that specifies, for every wealth level, the set of contracts that have been offered by entrants.[17] Let $\mathcal{M}$ denote the set of feasible menus of contracts. An *optimal allocation* $\alpha|\mu : [0, +\infty] \times \{L, H\} \times \mathcal{M} \to \mu(W) \cup \{\emptyset\}$ is a function that specifies, for every menu μ and every entrepreneur, at most one element from the set $\mu(W) \cup \{\emptyset\}$, where

$$\alpha_i(W|\mu) \in \underset{\psi \in \mu(W)}{\arg\max} \{\pi_i(X_i - R) + (W - S)(1 + r) : \pi_i(X_i - R) - S(1 + r) \geq 0\}$$

[12] This assumption is only for simplicity. One can allow wealth to be pledged as collateral without affecting the results. However, in that case, the set of feasible contracts must be slightly extended. An equivalent modelling strategy is to assume that agents are indifferent between consuming in the two periods and that the good is perishable.

[13] One could alternatively assume that the ex post return was costly verifiable as in Boyd and Smith (1993), who show that under adverse selection and costly state verification, debt contracts are optimal.

[14] See also Tirole (2006) for a discussion.

[15] Note also that, as each project requires a fixed amount of investment, if an entrepreneur signs a contract that requires her to invest S, the bank will invest exactly $I - S$.

[16] To see this, suppose instead that wealth is unobservable but that an entrepreneur can voluntarily reveal any part of it to a bank. Then, a loan contract should be written on the part of the wealth that was voluntarily revealed. This would require an additional variable to be specified, i.e., the part of the wealth that the entrepreneur decided to reveal to the bank. The results are the same under both modelling approaches.

[17] Note that for every wealth level, there might be several contracts offered in the market from which entrepreneurs can select.

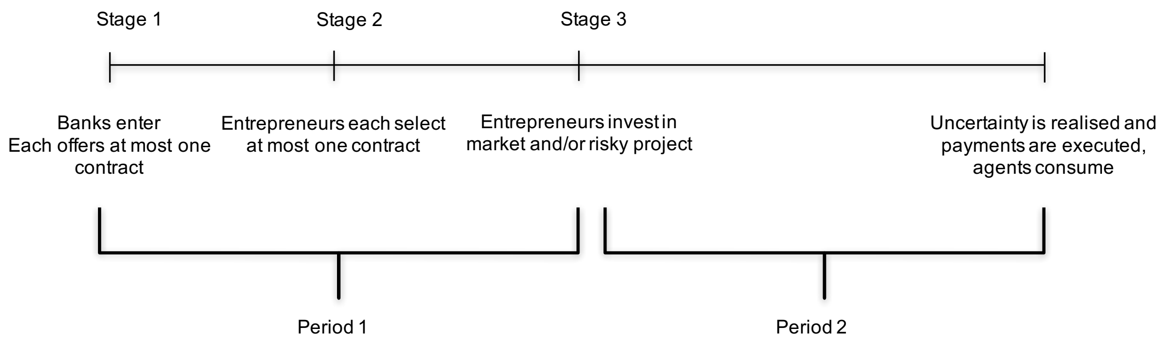


Figure 1: The timing of event - competitive screening

Because entrepreneurs have the choice not to undertake the risky project, in other words, when no contract offered by banks provides an entrepreneur a payoff greater than that she can earn by investing her wealth in the market, she does not borrow, and therefore, she is allocated the empty set.

Based on RS's seminal study of competitive screening markets, an equilibrium is defined below.

Definition 1. *An equilibrium consists of a menu of contracts and an optimal allocation* $(\hat{\mu}, \hat{\alpha})$ *such that (i) given* $\hat{\alpha}$*, no contract in the menu* μ *earns negative expected profits, and (ii) there exists no* W *and* $\tilde{\psi}$ *that, given* $\hat{\alpha}$*, if* $\tilde{\psi}$ *is included in* $\hat{\mu}(W)$*, it will earn strictly positive profits.*

The definition of equilibrium can be readily summarised as follows. Suppose that a menu of contracts is traded in the market, i.e., entrepreneurs each select at most one contract and/or invest any extra funds in the market rate. For the market to be in equilibrium, two conditions need to be satisfied. First, no contract from those offered in the market by incumbents yields negative profits to a bank. Second, no profitable opportunity for an entrant exists. In other words, no contract could be offered that could attract some entrepreneur and yield strictly positive profits.[18]

Although in RS it is well known that a free-entry equilibrium may not exist in some markets, this is not the case here thanks to Assumption 1.[19]

B. Characterisation of Equilibrium Investment

- *Equilibrium Contracts.* I solve the model through a series of propositions. After every result, I provide a discussion that explains the intuition behind it. I now state the first result regarding equilibrium investment.

Proposition 1. *Suppose that Assumption 1 is satisfied. If* $(\hat{\mu}, \hat{\alpha})$ *constitutes an equilibrium, then, for every* $W \in [0, +\infty]$, $\hat{\alpha}_L(W|\hat{\mu}) \in \emptyset$.

[18] One technical and conceptual difficulty regards the continuum of wealth levels. As is usual in large atomless economies, when a bank targets entrepreneurs of a certain wealth level, its expected profit is zero, as the measure of these entrepreneurs in the population is zero. One should think of the continuum as the limit case of a finite economy. Focusing on the continuum simplifies the comparative statics.

[19] Recall that, in RS, (i) when it exists, an equilibrium entails separation of types, and (ii) an equilibrium might not exist when the share of low-risk types in the population is high enough.

Proof. See Appendix A. □

In words, this result states that there exists no equilibrium in which a type-L entrepreneur invests in the risky project. The intuition behind this result is as follows. First, one can show that, for every possible wealth level W, no separating equilibrium exists in which a type-L entrepreneur with wealth W invests in the risky project. Therefore, if both types with wealth W invest in the risky project, the only possibility is that the repayment rate to the bank reflects the two types' weighted average probability to succeed. Assumption 2 rules out this possibility since, for any possible amount of wealth, this repayment rate is too high for a type-L entrepreneur to invest. Therefore, such an equilibrium is not sustainable.

In light of Proposition 1, the only possibility for the existence of an equilibrium in which some entrepreneurs invest in the risky project is the possibility that only type H invests by using enough of her initial wealth as skin in the game. This scenario is possible because wealth introduces an implicit cost for type-L entrepreneurs who possess a negative NPV project. This implicit cost allows banks to use wealth as a screening device to discourage type L from investing in the risky project. Nonetheless, because screening might be costly, even some type-H entrepreneurs might be unable to guarantee financing and instead be forced to invest in the market rate. Therefore, to characterise an equilibrium, one needs to find those type-H entrepreneurs who indeed invest in the risky project instead of the market rate. The following proposition provides a necessary and sufficient condition for a contract to belong to the equilibrium menu of contracts.

Proposition 2. *Suppose that Assumption 1 is satisfied; then, $(\hat{\mu}, \hat{\alpha})$ constitutes an equilibrium if and only if, for every $W \in [0, +\infty]$,*

$$\hat{\alpha}_H(W|\hat{\mu}) \in \underset{\psi \in [0,W] \times [0,X_H]}{\arg\max} \; \pi_H(X_H - R) + (W - S)(1 + r) \text{ subject to}$$

$$\pi_H R - (I - S)(1 + r) \geq 0 \tag{1}$$

$$\pi_L(X_L - R) - S(1 + r) \leq 0 \tag{2}$$

$$\pi_H(X_H - R) - S(1 + r) \geq 0 \tag{3}$$

In words, this proposition states that for any W, an equilibrium decision for type H entails either the empty set, which means investing in the market rate, or a loan contract that is optimal for type H within the set of contracts that satisfy the following requirements: (i) banks earn positive profits (i.e., Constraint (1)), (ii) a type-L entrepreneur with wealth W weakly prefers to invest in the market rate in lieu of the risky project (i.e., Constraint (2)), and (iii) a type-H entrepreneur with wealth W weakly prefers to invest in the risky project in lieu of the market rate (i.e., Constraint (3)). The allocation that is specified as an equilibrium allocation is similar to the *least-costly separating allocation* that features prominently in the seminal studies of markets with asymmetric information (e.g., Spence (1973), RS, Wilson (1977), Riley (1979), Spence (1978)). Note also that combining Propositions 1 and 2 implies that an equilibrium always exists.

The following proposition provides a necessary and sufficient condition for type H to invest in the risky project.

Proposition 3. *Suppose that Assumption 1 is satisfied; then, $(\hat{\mu}, \hat{\alpha})$ constitutes an equilibrium if and only if $\hat{\alpha}_H(W|\hat{\mu}) \notin \emptyset$ for every*

$$W \geq \overline{W}(r) \equiv \frac{\Delta X}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H}\right)(1+r)} \tag{4}$$

Proof. See Appendix A. □

Proposition 3 plays a key role in the analysis, stating that type-H entrepreneurs with insufficient wealth are unable to guarantee financing. The intuition behind this result can be readily summarised as follows. For a bank to discourage a type-L entrepreneur from investing in the risky project, it needs to sufficiently raise the repayment rate. Because the repayment rate is inversely related to how much skin in the game an entrepreneur has and because $X_H < X_L$, only relatively "wealthy" entrepreneurs can afford to undertake a loan. All type-H entrepreneurs with insufficient wealth are instead denied credit.

Let $\psi_H(W) = (S_H(W), R_H(W))$ denote the equilibrium contract of a type-H entrepreneur with wealth W. As shown in the proof of Proposition 3, for every $\overline{W}(r) \leq W < \overline{\overline{W}}(r)$, where

$$\overline{\overline{W}}(r) \equiv \frac{X_L - \frac{I(1+r)}{\pi_H}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H}\right)(1+r)}$$

Constraint (2) is binding and $S_H(W) = W$. Therefore, $R_H(W) = X_L - \frac{W(1+r)}{\pi_L}$. The intuition is that $R_H(W) = X_L - \frac{W(1+r)}{\pi_L}$ is the minimum possible repayment rate a bank can charge to discourage a type-L entrepreneur with wealth W from borrowing. This sort of *limit pricing* has two significant implications. First, a bank that contracts with a type-H entrepreneur with wealth $\overline{W}(r) \leq W < \overline{\overline{W}}(r)$ earns strictly positive profits. Second, as explained above, $R_H(W) = X_L - \frac{W(1+r)}{\pi_L}$ is strictly decreasing in the wealth of an entrepreneur. In other words, a type-H entrepreneur's share of the ex post project surplus is strictly decreasing in her wealth. This will have significant implications when the model is extended to accommodate endogenous projects and moral hazard.[20]

- *Aggregate Investment and Comparative Statics.* It is evident that the minimum amount of wealth that is required by banks as skin in the game is a decreasing function of r.

[20]To complete the characterisation of equilibrium, note that for every for every $W \geq \overline{\overline{W}}(r)$, $\overline{\overline{W}}(r) \leq S_H(W) \leq W$ and $R_H(W) = \frac{(I-S)(1+r)}{\pi_H}$. In other words, when entrepreneurs are sufficiently wealthy, they pay an actuarily fair repayment rate and the level of own-financing is indeterminate.

This has an intuitive explanation. An increase in r has two countervailing effects for a type-H entrepreneur. On the one hand, it increases her opportunity cost of investing in the risky project. On the other hand, because it also increases the opportunity cost of a type-L entrepreneur to invest in the risky project and because type L's opportunity cost is strictly higher than type H's opportunity cost, there is a decrease in the repayment rate of borrowing and investing in the risky project for every unit of wealth. The latter effect always dominates the former, and hence, even entrepreneurs with a lower amount of wealth now find it profitable to invest in the risky project. I call this effect the *screening effect* of interest rates because a change in the interest rate either facilitates or hinders banks' efforts to effectively screen high- from low-quality projects.

Aggregate investment as a function of the market rate is given by

$$AI(r) = \lambda_H I[1 - F(\overline{W}(r))]$$

Therefore, aggregate investment is strictly increasing in r because the lower bound of the cutoff defined in Eq. (4) is strictly decreasing in r. Moreover, an increase in λ_H or π_H or a shift in the distribution of wealth in the sense of first order stochastic dominance all shift the aggregate investment curve to the left. In contrast, an increase in ΔX or π_L shifts the investment curve to the right. In other words, an increase in risk reduces investment.

3 Project Choice and Moral Hazard

A. Extending the Model

- *Entrepreneurs and Projects.* In this section, I extend the model to account for endogenous project choice by entrepreneurs. There are two types of privately informed entrepreneurs endowed with an initial amount of wealth as in Section 2. I continue to assume that i and W are independently distributed. There are two potential projects. The *baseline project* requires an investment of I to commence and returns X_i with probability π_i^b, whereas with probability $1 - \pi_i^b$, it returns zero. Alternatively, an entrepreneur can undertake an *advanced project* that also requires an investment of I to commence but returns X_i with probability π_i^a, whereas with probability $1 - \pi_i^a$, it returns zero. For a type-i entrepreneur, the cost of running project $j \in \{a, b\}$ is denoted by c_i^j, where $c_L^a = c_L^b = c_H^b = 0$ and $c_H^a = c > 0$. The positive cost of running the advanced project for a type-H entrepreneur reflects, perhaps, the additional effort that the entrepreneur needs to devote to it. For notational simplicity, let $\Delta X = X_L - X_H$ and $\Delta\pi_i = \pi_i^a - \pi_i^b$.

The projects satisfy the following assumptions:

Assumption 2. *(i)* $\pi_L^b = \pi_L^a = \pi_L$, *(ii)* $\Delta X > 0$, *(iii)* $\pi_L X_L < \pi_H^b X_H < \pi_H^a X_H$, *(iv)* $X_H > \max\left\{\frac{I(1+r)}{\pi_H^a} + \frac{c}{\pi_H^a}, \frac{c}{\Delta\pi_H}\right\}$, *and, (v)* $X_L < \min\{\frac{I(1+r)}{\lambda_L \pi_L + \lambda_H \pi_H^a}, X_H + \left(\frac{\pi_H^a}{\pi_L} - 1\right)\frac{c}{\Delta\pi_H}\}$

According to Condition (i) of Assumption 2, type-L entrepreneurs have essentially only one project available, as described in the previous section. This assumption is only for simplicity because one could accommodate more than one project for type L as long as both projects are of strictly negative NPV.[21] Type-H entrepreneurs have two distinct

[21] Although it initially appears redundant to assume that type-L entrepreneurs have two projects, as both projects are payoff-equivalent, this simplifies the definition of equilibrium. See below.

projects. Conditions (i), (ii) and (iii) of Assumption 2 combined imply that all projects can be ranked based on risk. In particular, as in Assumption 1, both of type H's projects second-order stochastically dominate type L's project. Moreover, type H's advanced project first-order stochastically dominates her baseline project.

Condition (iv) of Assumption 2 has a dual implication. First, it implies that type H's advanced project is of positive NPV. Second, it is superior (i.e., yields higher net expected return) than the baseline project. Part (v) implies that absent wealth, the financial market collapses, which gives fundamental scope to wealth. In fact, as I show below, Part (v) is sufficient for the existence of equilibria with interesting properties. Requiring that $X_L < X_H + \left(\frac{\pi_H^a}{\pi_L} - 1\right)\frac{c}{\Delta\pi_H}$ allows the equilibrium cutoffs to be well behaved as functions of the interest rate and hence is a regularity assumption.

- *Banks, Contracts and Competition.* As in Section 2, I restrict attention to simple risky-debt contracts of the form $(S, R) \in \mathbb{R}_+^2$. The timing of events is as specified in Section 2 with one significant difference. In the last stage of period one, entrepreneurs who borrowed from a bank decide which project to undertake (i.e., baseline or advanced), whereas those who did not do so invest in the market rate. Banks are unable to observe the choice of project, and hence, alongside the adverse selection problem, a moral hazard problem might arise. The timing of events is summarised in Figure 2

A menu of contracts $\mu : [0, +\infty] \twoheadrightarrow [0, I] \times [0, X_H]$ is as defined in Section 2. However, an *optimal allocation* $\alpha : [0, +\infty] \times \{L, H\} \times \mathcal{M} \to \{\mu(W) \times \{b, a\}\} \cup \{\emptyset\}$ is a function that specifies, for every menu μ and every entrepreneur, at most one element from the set $\{\mu(W) \times \{b, a\}\} \cup \{\emptyset\}$, where

$$\alpha_i(W|\mu) \in \underset{(\psi,j)\in\{\mu(W)\}\times\{b,a\}}{\arg\max} \{\pi_i^j(X_i - R) + (W - S)(1 + r) - c_i^j : \pi_i^j(X_i - R) - S(1 + r) - c_i^j \geq 0\}$$

Unlike Section 2, entrepreneurs have three available choices: (i) undertaking the baseline project, (ii) undertaking the advanced project, or (iii) investing in the market rate. When no contract offered by banks provides an entrepreneur a payoff greater than that from investing her wealth in the market, she does not borrow, and therefore, she is allocated the empty set.

The definition of equilibrium is specified in Definition 1.

B. Characterisation of Equilibrium Investment

- *Preliminaries.* First, one can show that there exists no equilibrium in which type L borrows and invests in the risky project. The proof of this proposition in similar to that provided for Proposition 1 and is therefore omitted. It relies on Assumption 1, particularly that $X_L < \frac{I(1+r)}{\lambda_L\pi_L + \lambda_H\pi_H^a}$.

Proposition 4. *Suppose that Assumption 2 is satisfied. If $(\hat{\mu}, \hat{\alpha})$ constitutes an equilibrium, then, for every $W \in [0, +\infty]$, $\hat{\alpha}_L(W|\hat{\mu}) \in \emptyset$.*

In light of Proposition 4, the question of interest is to characterise which type-H entrepreneurs, if any, invest and in which project. However, unlike Section 2, banks need not

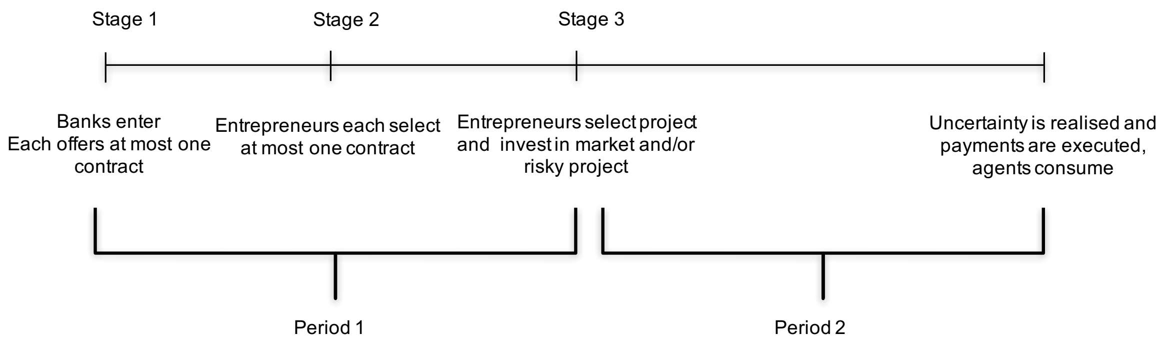


Figure 2: The timing of events - competitive screening and moral hazard

only to discourage type-L entrepreneurs from investing in the risky project but also, due to the moral hazard problem, to provide appropriate incentives to type-H entrepreneurs to invest in the advanced project. Therefore, there is an interplay among screening, moral hazard and type H's participation constraints. Finding which constraints are binding is key to characterising an equilibrium.

- *Equilibrium Contracts.* In accord with Section 2, key to the characterisation of an equilibrium is the linear program that maximises the payoff of type-H entrepreneurs subject to a set of incentive-compatibility, individual-rationality and feasibility constraints. The following proposition is in the spirit of Proposition 2.

Proposition 5. *Suppose that Assumption 2 is satisfied; then, $(\hat{\mu}, \hat{\alpha})$ constitutes an equilibrium if and only if, for every $W \in [0, +\infty]$,*

$$\hat{\alpha}_H(W|\hat{\mu}) \in \underset{(\psi,j)\in[0,W]\times[0,X_H]\times\{b,a\}}{\arg\max} \pi_H^j(X_H - R) + (W - S)(1+r) - c_i^j \text{ subject to}$$

$$\pi_H^j R - (I - S)(1+r) \geq 0 \tag{5}$$

$$\pi_L(X_L - R) - S(1+r) \leq 0 \tag{6}$$

$$\pi_H^j(X_H - R) - S(1+r) - c_H^j \geq 0 \tag{7}$$

A notable difference between the programs specified in Propositions 2 and 5 is the choice of the project by a type-H entrepreneur combined with a contract offered by a bank. Because banks compete in loan contracts, only contracts that provide the highest payoff to a type-H entrepreneur are sustainable as equilibrium contracts. The proof of Proposition 5 is similar to that of Proposition 2 and hence is omitted.

The question of interest now becomes the characterisation of the solution of the linear program given in Proposition 5. The two conditions that seem crucial are (i) whether the baseline project is of positive NPV, and (ii) whether screening allows the provision of incentives to type-H entrepreneurs to undertake the advanced project. The following proposition is the first step towards the characterisation of an equilibrium.

Proposition 6 (Severe Adverse Selection)**.** *Suppose that Assumption 2 is satisfied and* $X_L > \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$.

If $X_H \geq \frac{I(1+r)}{\pi_H^b}$, *then* $(\hat{\mu}, \hat{\alpha})$ *constitutes an equilibrium if and only if*

(i) $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$ *for every* $W < W_1(r) \equiv \frac{\Delta X}{(\frac{1}{\pi_L} - \frac{1}{\pi_H^b})(1+r)}$

(ii) $\hat{\alpha}_H(W|\hat{\mu}) \in \{(m, b) : m \in \hat{\mu}(W)\}$ *for every* $W_1(r) \leq W \leq W_2(r) \equiv \frac{\Delta X + \frac{c}{\Delta\pi_H}}{\frac{1+r}{\pi_L}}$

(iii) $\hat{\alpha}_H(W|\hat{\mu}) \in \{(m, a) : m \in \hat{\mu}(W)\}$ *for every* $W_2(r) \leq W \leq +\infty$

If $X_H < \frac{I(1+r)}{\pi_H^b}$, *then* $(\hat{\mu}, \hat{\alpha})$ *constitutes an equilibrium if and only if*

(i) $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$ *for every* $W < W_2(r)$

(ii) $\hat{\alpha}_H(W|\hat{\mu}) \in \{(m, a) : m \in \hat{\mu}(W)\}$ *for every* $W_2(r) \leq W \leq +\infty$

Proof. See Appendix A. □

Proposition 6 specifies those conditions that are sufficient to make adverse selection a severe problem in the financial market. Intuitively, this occurs when the ex post return of a type-L entrepreneur is relatively high. In such a case, banks need to raise their lending standards sufficiently to discourage type-L entrepreneurs from acquiring loans. This introduces two potential inefficiencies. If the baseline project of type-H entrepreneurs is of positive NPV, then there is a positive wealth threshold below which no entrepreneur invests. Therefore, as in Section 2, some entrepreneurs are unable to access the financial market. For intermediate wealth levels, type-H entrepreneurs undertake the baseline project, whereas only wealthy type-H entrepreneurs can afford to undertake the advanced project. Therefore, even if type-H entrepreneurs would have undertaken the advanced project had type-L entrepreneurs not existed, the presence of type-L entrepreneurs introduces such a high negative externality to type-H entrepreneurs that prevents the latter from undertaking the advanced project.

I now characterise the equilibrium when some of the hypotheses of Proposition 6 are not satisfied.

Proposition 7 (Severe Moral Hazard)**.** *Suppose that Assumption 2 is satisfied and* $X_L \leq \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$.

If $X_H \geq \frac{I(1+r)}{\pi_H^b}$, *then* $(\hat{\mu}, \hat{\alpha})$ *constitutes an equilibrium if and only if*

(i) $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$ *for every* $W < W_1(r)$

(ii) $\hat{\alpha}_H(W|\hat{\mu}) \in \{(m, b) : m \in \hat{\mu}(W)\}$ *for every* $W_1(r) \leq W \leq W_3(r) \equiv I - \frac{\pi_H^a}{1+r}\left(X_H - \frac{c}{\Delta\pi_H}\right)$

(iii) $\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,a) : m \in \hat{\mu}(W)\}$ *for every* $W_3(r) \leq W \leq +\infty$

If $X_H < \frac{I(1+r)}{\pi_H^b}$*, then* $(\hat{\mu}, \hat{\alpha})$ *constitutes an equilibrium if and only if*

(i) $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$ *for every* $W < W_3(r)$

(ii) $\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,a) : m \in \hat{\mu}(W)\}$ *for every* $W_3(r) \leq W \leq +\infty$

Proof. See Appendix A. □

Unlike Proposition 6, Proposition 7 specifies those conditions that are sufficient to make moral hazard a severe problem in the financial market. This occurs when the ex post return of type-L entrepreneurs is not sufficiently high. In such a case, banks can more easily discourage type-L entrepreneurs from acquiring a loan. However, it is now more difficult to provide incentives to type-H entrepreneurs to undertake the advanced project. The equilibrium is characterised, potentially, by two margins. If the baseline project is of positive NPV, then there is a positive measure of type-H entrepreneurs who are unable to access the financial market. For intermediate wealth levels, type-H entrepreneurs invest in the baseline project. It is only for sufficiently high wealth levels that entrepreneurs can invest in the advanced project. If the baseline project is of negative NPV, then only for sufficiently high wealth levels do type-H entrepreneurs acquire funds and invest in the advanced project.

- *Aggregate Investment and Comparative Statics.* Based on Propositions 6 and 7, one can characterise the aggregate investment correspondence and its composition (i.e., the shares of advanced and baseline projects). Let $AI(r)$, $I^b(r)$ and $I^a(r)$ denote the aggregate, baseline and advanced investment correspondences, respectively.

Corollary 1. *(i) If* $X_L > \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$ *and* $X_H \geq \frac{I(1+r)}{\pi_H^b}$*, then*

$$\frac{\partial AI(r)}{\partial r} > 0$$

$$\frac{\partial I^b(r)}{\partial r} \textit{ ambiguous}$$

$$\frac{\partial I^a(r)}{\partial r} > 0$$

(ii) If $X_L > \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$ *and* $X_H < \frac{I(1+r)}{\pi_H^b}$*, then*

$$\frac{\partial AI(r)}{\partial r} > 0$$

$$\frac{\partial I^b(r)}{\partial r} = 0$$

$$\frac{\partial I^a(r)}{\partial r} > 0$$

(iii) If $X_L \leq \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$ *and* $X_H \geq \frac{I(1+r)}{\pi_H^b}$*, then*

$$\frac{\partial AI(r)}{\partial r} > 0$$

$$\frac{\partial I^b(r)}{\partial r} > 0$$

$$\frac{\partial I^a(r)}{\partial r} < 0$$

(iv) If $X_L \leq \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$ *and* $X_H < \frac{I(1+r)}{\pi_H^b}$*, then*

$$\frac{\partial AI(r)}{\partial r} < 0$$

$$\frac{\partial I^b(r)}{\partial r} = 0$$

$$\frac{\partial I^a(r)}{\partial r} < 0$$

Proof. See Appendix A. □

In all but Case (iv) of Corollary 1, $\partial AI(r)/\partial r < 0$. In Case (iv), $\partial AI(r)/\partial r > 0$. However, the effect of a change in the interest rate on the composition of investment is not as clear. In Cases (i)-(ii), an increase in r increases investment in advanced projects. Nonetheless, in Case (i), an increase in r has an ambiguous effect on investment in baseline projects. This has the following explanation. Although the marginal borrower decreases, more entrepreneurs find it profitable to invest in advanced projects. Therefore, if the distribution of wealth is not skewed to the right, an increase in r might cause a decrease in baseline projects.[22] In Case (iii), an increase in r causes an increase in investment in baseline projects and a decrease in investment in advanced projects.

C. A Numerical Example

In this section, I provide a numerical example that illustrates the results. Consider the following parameter values:

With these parameter values, one obtains the following cutoffs:

$$\frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right) = 50(1+r) - 39$$

$$\frac{I(1+r)}{\pi_H^b} = \frac{50(1+r)}{3}$$

Therefore, by applying Propositions 6 and 7, $AI(r)$, $I^b(r)$ and $I^a(r)$ are determined as follows:

[22] See the numerical example in the next section.

$c = 1$
$I = 10$
$X_H = 18$
$X_L = 19$
$\pi_H^b = 0.6$
$\pi_H^a = 0.8$
$\pi_L = 0.2$
$\lambda_H = 0.5$
$W \sim$ LN$(0.275, 0.125)$

Table 1

$$AI(r) = \begin{cases} 5[1 - F(0.3/(1+r))], & \text{if } 0 \leq r \leq 0.08 \\ 5[1 - F(1.2/(1+r))], & \text{if } 0.08 < r \leq 0.16 \\ 5[1 - F(10 - 10.4/(1+r))], & \text{if } 0.16 < r \leq 0.34 \\ 0, & \text{otherwise} \end{cases}$$

$$I^b(r) = \begin{cases} 5[F(1.2/(1+r)) - F(0.3/(1+r))], & \text{if } 0 \leq r \leq 0.08 \\ 0, & \text{otherwise} \end{cases}$$

$$I^a(r) = \begin{cases} 5[1 - F(1.2/(1+r))], & \text{if } 0 \leq r \leq 0.16 \\ 5[1 - F(10 - 10.4/(1+r))], & \text{if } 0.16 < r \leq 0.34 \\ 0, & \text{otherwise} \end{cases}$$

The investment curves are depicted in Figure 3. Note that for any interest rate below 8%, the investment curve in baseline projects is downward sloping and drops to zero for any interest rate above 8%. Investment in advanced projects has an inverse-U shape. It increases up to 16% and decreases for any interest rate above 16% until it drops to zero at an interest rate of 34%.

4 Discussion

The model has provided a bare-bones illustration of the effect of (safe) interest rate changes on aggregate investment and its composition under competitive screening and moral hazard. The model emphasised the significance of entrepreneurial wealth (i.e., skin in the game) in facilitating competitive banks' efforts to effectively screen positive from negative NPV projects and alleviating the potential moral hazard problem. Among others, the model has several implications on the effect of monetary policy on stimulating investment.

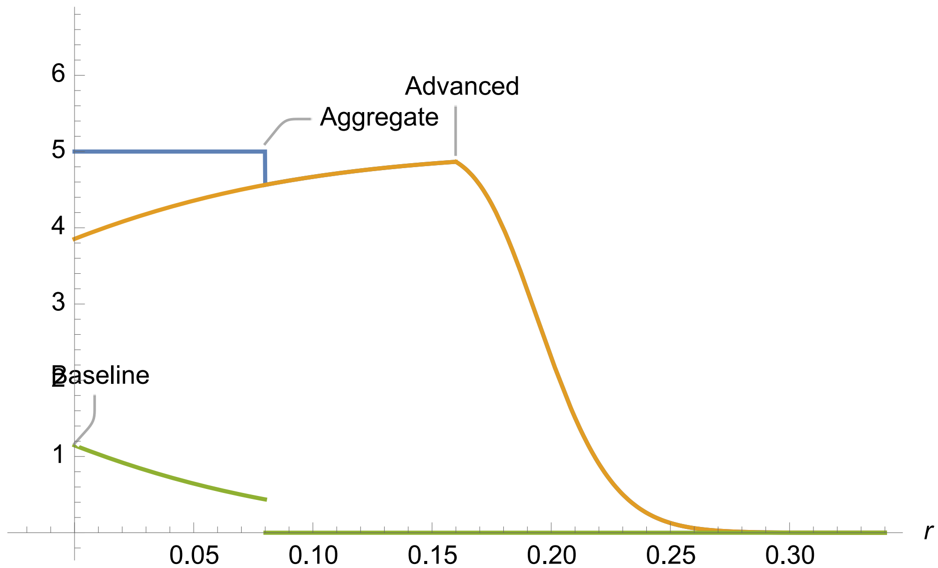
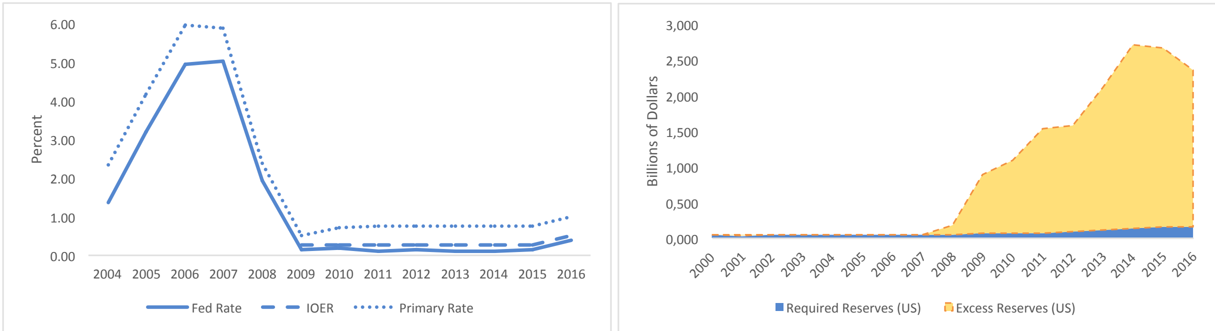


Figure 3

- **Implication 1**: *Ultra-low interest rates might have an adverse impact on investment*. Economists have raised concerns about prolonged periods of low interest rates (see, among others, Rajan (2005), Taylor (2007), Borio and Zhu (2012) and Summers (2014)). Rajan (2011) argues that a prolonged period of near-zero interest rate policy can lead to market distortions and asset price inflation. A considerable amount of empirical literature finds evidence that during the period that preceded the financial crisis, there was indeed a deterioration of lending standards and over-leverage by large financial institutions (see, among others, Keys et al. (2012) and Adrian and Shin (2010)).[23] However, this argument fails to explain the stylised facts documented in the aftermath of the financial crisis, when central banks decreased interest rates to unprecedentedly low levels but depository institutions were nevertheless reluctant to expand credit. For instance, to avoid a complete meltdown, the Fed reduced its target rates to near zero (see Figure 4a). However, the decline in interest rates was accompanied by an explosion of excess reserves by depository institutions far above those required by regulatory standards (see Figure 4b). This paper provides a novel insight by showing that low interest rates might in fact tighten banks' lending standards and therefore lead to stagnant investment. This might be more prevalent in the aftermath of financial crises when banks become more risk averse, the cdf of wealth shifts upwards, or the likelihood of default increases.

Figure 5 plots the federal funds rate along with the growth rate of commercial and industrial loans granted from US commercial banks from 1955 to 2016. Except in the 1980-1982 period, these two rates seem to follow a surprisingly similar pattern: low rates are accompanied by low growth and high rates by high growth. In the aftermath of the crisis, the federal funds rate has been approximately 0.5%, whilst the growth rate of commercial loans has been approximately 10%, as high as it was during the second half of the 1980s

[23] Dell'Ariccia, Laeven, and Marquez (2014) and Bolton, Santos, and Scheinkman (2016b), building on Bolton, Santos, and Scheinkman (2016a) and Martinez-Miera and Repullo (2017), provide microfoundations for this argument.



(a) (b)

Figure 4: Panel (a) depicts the Fed policy rates for the period 2004-2016. Panel (b) depicts the required and excess reserves for the period 2000-2016. **Source:** Federal Reserve Bank of St. Louis (FRED)

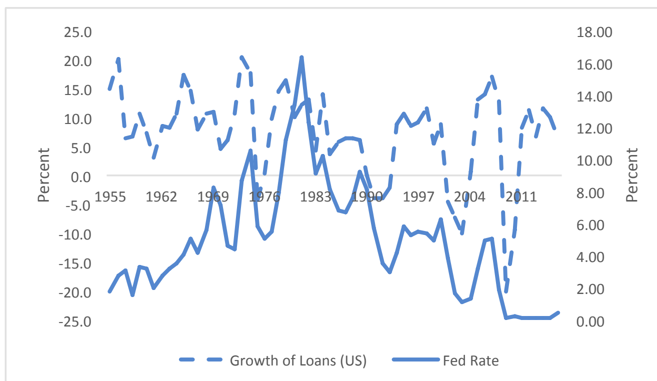
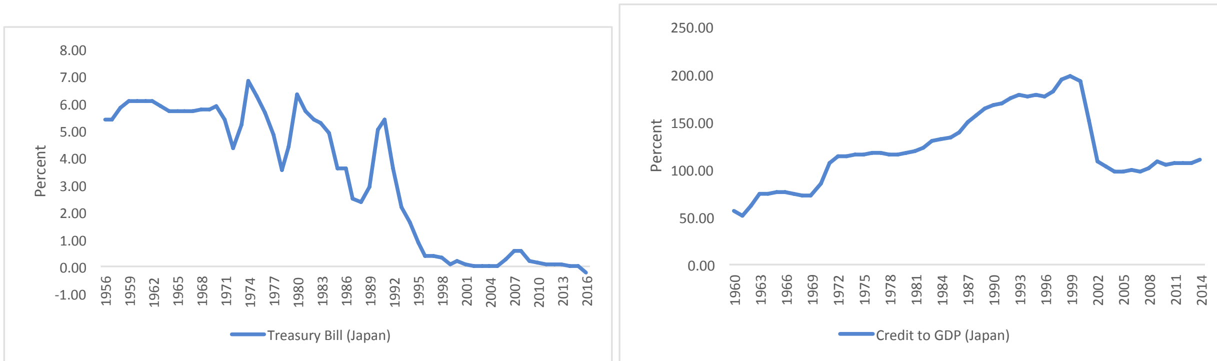


Figure 5: This graph depicts the growth rate of commercial and industrial loans granted by US depository institutions alongside the (annual) federal funds rate. These two seem to follow a surprisingly similar pattern. **Source:** Federal Reserve Bank of St. Louis (FRED)

and 1990s, when the federal funds rate was approximately 6.5% and 5.5%, respectively.

The US experience was not entirely new. In 1995, Japan suffered a severe recession after a decade of weak productivity growth. The central bank of Japan (BoJ) responded by expanding monetary policy and engaging in broad purchases of long-term assets (i.e., quantitative easing) to lower long-term rates. These actions led to an unprecedented increase in the monetary base and, in line with the current US experience, an explosion of excess reserves by depository institutions. Nonetheless, the desired increase in lending and investment never followed, and Japan has since faced several subsequent recessions.[24] In line with the model's predictions, the ratio of credit to GDP by financial

[24] Krugman (1998, 2000) argues that Japan caught itself in a liquidity trap (see Keynes (1936) and Minsky (2008)). A liquidity trap refers to a situation in which the monetary authority loses effective control of interest rates. Excessive liquidity lowers interest rates to the extent that economic agents prefer to hold cash rather than bonds. In other words, money and bonds become perfect substitutes. This traditional perspective neglects the role of financial intermediaries in the financial system and focuses primarily on a demand-driven explanation for weak investment. In this paper, I emphasise the role of financial intermedi-



(a) (b)

Figure 6: Panel (a) depicts the three-month treasury bill interest rate of the Japanese government. Since 1995, short-term interest rates in Japan have been close to zero or become negative. Panel (b) depicts the volume of credit as a percentage of GDP. At the onset of the 1995 recession, there was a significant decline that has lasted to the present. **Source:** Federal Reserve Bank of St. Louis (FRED)

institutions fell to half its 1995 peak and has remained at this level to date (see Figure 6a), whilst nominal and real short-term rates have remained close to zero for almost two decades (see Figure 6b).

- **Implication 2**: *More investment is not necessarily better.* The prevalent viewpoint is that frictions in financial markets lead to insufficient investment. For instance, in Jaffee and Russell (1976) and SW, banks prefer to ration credit to entrepreneurs even if the latter are willing to pay a higher interest rate, and in Myers and Majluf (1984), firms may avoid issuing new equity to undertake new projects due to potential underpricing. In HT, banks ration credit to undercapitalised firms to ensure that funds will end up in high-quality projects. De Meza and Webb (1987) challenge the findings of SW by showing that over-investment may arise in equilibrium.

The model also implies that investment is insufficient in equilibrium, as banks prefer to provide no credit to low-capitalised firms instead of lowering their loan-repayment rates. Nonetheless, the model offers a novel insight related to the composition of investment. In particular, not all entrepreneurs who acquire funds invest in high-quality projects, as banks might sufficiently raise their rates to discourage entrepreneurs of low-quality projects from accessing the market. This increase comes at a cost, as it leaves insufficient incentives to entrepreneurs of high-quality projects to invest in superior projects. As shown in the numerical example, an increase in the interest rate, may lead not only to higher investment but to more investment in advanced projects (see Figure 3).

- **Implication 3**: *As interest rates rise, entrepreneurs invest in safer projects.* As discussed in *Implication 1*, there is a widespread concern among scholars and policy makers that a prolonged period of low interest rates led to a deterioration of lending standards, which

aries as loan originators and provide a supply-driven explanation for insufficient lending and investment.

was followed by a credit boom in subprime mortgages. This argument has given rise to a heated debate among economists over whether the Fed contributed to the financial crisis with a "too-lax-for-too-long" monetary policy.[25] The empirical identification of the *risk-taking channel* of monetary policy has attracted considerable attention in the literature. Jiménez et al. (2014) analyse an exhaustive dataset of firm loans from Spain and find that a prolonged period of short-term (i.e., overnight) interest rates had a positive effect on the volume of credit to riskier firms with the effect being more pronounced for low-capitalised banks. Dell'Ariccia, Laeven, and Suarez (2017) confirm the former effect in US data but find that this was less pronounced for low-capitalised banks.

The model illustrated that lower interest rates may indeed affect the composition of investment by inducing more lending to baseline projects, which are by definition riskier. In that sense, the asset-side of a bank's balance sheet might become riskier. Nonetheless, this is not necessarily due to deterioration of lending standards but rather to the conflict between competitive screening and incentive provision to healthy firms to invest in safer projects.

Beyond monetary policy implications, the model has a number of testable implications concerning the relationship among indebtedness, likelihood of default and the repayment to debt ratio.

- **Implication 4**: *Among firms with similar observable characteristics, the (loan) repayment to debt ratio is increasing in indebtedness.* A main lesson from the analysis is that low wealth hinders the ability of banks to perform effective screening. To discourage low-type entrepreneurs from undertaking the risky project, banks need to sufficiently raise the repayment rate. This might create an inverse relationship between the ratio of the repayment rate and indebtedness.

Formally, the ratio of repayment to debt is given by

$$q = \frac{R}{I - S} \tag{8}$$

As shown in Section 2, for every $\overline{W}(r) \leq W < \overline{\overline{W}}(r)$, $S_H(W) = W$ and $R_H(W) = X_L - \frac{W(1+r)}{\pi_L}$, whereas for every $W \geq \overline{\overline{W}}(r)$, $\overline{\overline{W}}(r) \leq S_H(W) \leq W$ and $R_H(W) = \frac{(I - S_H(W))(1+r)}{\pi_H}$. Therefore, Eq. (8) has the following form:

$$q(W) = \begin{cases} \frac{X_L - \frac{W(1+r)}{\pi_L}}{I - W}, & \text{if } \overline{W}(r) \leq W < \overline{\overline{W}}(r) \\ \\ \frac{1+r}{\pi_H}, & \text{if } W \geq \overline{\overline{W}}(r) \end{cases}$$

which due to Assumption 1 is strictly decreasing for $\overline{W}(r) \leq W < \overline{\overline{W}}(r)$ and constant for $W \geq \overline{\overline{W}}(r)$.

[25] See, for instance, Taylor (2007), Greenspan (2009) and Bernanke (2010).

- **Implication 5**: *When interest rates are low, among firms with similar observable characteristics, the share of defaulting firms is increasing in indebtedness.* Section 3 illustrated how competitive screening might hinder the ability of banks to provide incentives to entrepreneurs to undertake superior projects. This was more profound when interest rates were low, as in this case, screening imposes a higher cost. In particular, low-wealth entrepreneurs (who are hence more indebted) had insufficient incentives to undertake projects with a higher probability of success. This implies a positive relationship between the share of default and indebtedness.

5 Conclusion

In this paper, I studied a model of investment under competitive screening and moral hazard. I showed that entrepreneurial wealth serves an important role in alleviating asymmetric information and providing incentives to entrepreneurs to investing in high-quality projects. The model illustrated the potential for a hump-shaped investment curve. I discussed policy and testable implications of the model.

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APPENDIX A

■ **Proof of Proposition 1:** Suppose first that there exists an equilibrium $(\hat{\mu}, \hat{\alpha})$ and some W such that $\hat{\alpha}_L(W|\hat{\mu}) \neq \hat{\alpha}_H(W\hat{\mu})$ and $\hat{\alpha}_L(W|\hat{\mu}) \notin \emptyset$, $\hat{\alpha}_H(W|\hat{\mu}) \notin \emptyset$. Let $\hat{\psi}_i(W) = (\hat{S}_i(W), \hat{R}_i(W)) \equiv \hat{\alpha}_i(W|\hat{\mu})$. Condition (2) of Definition 1 imposes that $\hat{R}_i(W) = \frac{(I-\hat{S}_i(W))(1+r)}{\pi_i}$ for every i. The payoff of type L from contract $(\hat{S}_L(W), \hat{R}_L(W))$ is $\pi_L X_L - I(1+r) + W(1+r)$. In equilibrium, this payoff needs to be weakly greater than $W(1+R)$, which contradicts Assumption 1.

Suppose now that there exists an equilibrium $(\hat{\mu}, \hat{\alpha})$ such that $\hat{\alpha}_L(W|\hat{\mu}) = \hat{\alpha}_H(W\hat{\mu}) \notin \emptyset$. Let $\hat{\psi}(W) = (\hat{S}(W), \hat{R}(W)) \equiv \hat{\alpha}_i(W|\hat{\mu})$. Condition (ii) of Definition 1 imposes that $\hat{R}(W) = \frac{(I-\hat{s}(W))(1+r)}{\overline{\pi}}$. Consider the payoff of type L from contract $(\hat{S}(W), \hat{R}(W))$:

$$\pi_L X_L - \frac{\pi_L}{\overline{\pi}} I(1+r) + \Big(\frac{\pi_L}{\overline{\pi}} - 1\Big)\hat{S}(W)(1+r) + W(1+r) \tag{A.1}$$

Because the payoff of a type-L entrepreneur with wealth W from investing in the market rate is $W(1+r)$, the payoff given in Eq. (A.1) has to be weakly greater than $W(1+r)$, which means that either $\pi_L > \overline{\pi}$, $X_L - \frac{1}{\overline{\pi}} Ir > 0$, or both. However, we know by definition that $\pi_L < \overline{\pi}$ and, from Assumption 1, that $X_L - \frac{1}{\overline{\pi}} Ir < 0$. Hence, we have a contradiction. Q.E.D.

□ **Proof of Proposition 2:** I first prove the "if" part. Suppose that for some W, $\hat{\alpha}_H(W|\hat{\mu})$ belongs to the the set of maximisers of the linear program specified in Proposition 2 but does not belong to the equilibrium set. If $\hat{\psi}_H(W) = (\hat{S}_H(W), \hat{R}_H(W)) \equiv \hat{\alpha}_H(W|\hat{\mu}) \notin \emptyset$, one of the following is true:

(i) $\pi_H \hat{R}_H(W) - (I - \hat{S}_H(W))(1+r) < 0$,

or

(ii) there exists a contract $\tilde{\psi}$ that, given $\hat{\alpha}$, if included in $\hat{\mu}(W)$ will earn strictly positive profits.

Statement (i) is false because it immediately contradicts Constraint (1). If $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$, then only Statement (ii) can be true. Suppose that it is true. This implies that

$$\pi_H(X_H - \tilde{R}) + (W - \tilde{S})(1+r) > V_H(\hat{\psi}_H(W)|W)$$

where $V_H(\hat{\psi}_H(W)|W)$ is the payoff of type H with wealth W from $\hat{\psi}_H(W)$. Because $\tilde{\psi}$ provides a strictly higher payoff to type H with wealth W and satisfies Constraints (1) and (3), it is true that Constraint (2) is not satisfied, or

$$\pi_L(X_L - \tilde{R}) + (W - \tilde{S})(1 + r) > W(1 + r)$$

This contradicts Proposition 1. A similar argument establishes the result if $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$

I now prove the "only if" part. Suppose that for some W, $\hat{\alpha}_H(W|\hat{\mu})$ belongs to the equilibrium set but does not belong to the set of the maximisers of the linear program specified in Proposition 2. One of the following is true: $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$ or $\hat{\alpha}_H(W|\hat{\mu}) \notin \emptyset$. The following lemma is useful.

Lemma 1. *If $\hat{\psi}_H(W) = (\hat{S}_H(W), \hat{R}_H(W)) \equiv \hat{\alpha}_H(W|\hat{\mu}) \notin \emptyset$, then $\hat{\psi}_H(W)$ satisfies Constraints (1)-(3).*

Proof. Suppose that Constraint (1) is not satisfied; then, there is a contradiction with Condition (i) of Definition 1. Suppose that Constraint (2) is not satisfied; then, there is a contradiction with the definition of $\hat{\alpha}$. Suppose that Constraint (3) is not satisfied; then, there is a contradiction with Proposition 1. □

Suppose that the set of the maximisers of the linear program is the empty set, whereas $\hat{\alpha}_H(W|\hat{\mu}) \notin \emptyset$. Let the payoff of type H with wealth W from contract $\hat{\alpha}_H(W|\hat{\mu}) \equiv \hat{\psi}_H(W)$ be denoted by $V_H(\hat{\psi}_H(W)|W)$. From the definition of $\hat{\alpha}$, $V_H(\hat{\psi}_H(W)|W) \geq W(1 + r)$. Because the set of the maximisers of the linear program is the empty set, $\hat{\psi}_H$ needs to violate at least one of the Constraints (1)-(3). This contradicts Lemma 1.

Suppose now that the set of the maximisers of the linear program is not the empty set. Denote one of the maximisers by $\tilde{\psi}_H(W) = (\tilde{S}_H(W), \tilde{R}_H(W))$. Consider first the case, in which $\hat{\alpha}_H(W|\hat{\mu}) \equiv \hat{\psi}_H = (\hat{S}_H(W), \hat{R}_H(W)) \notin \emptyset$. Because $\hat{\psi}_H(W)$ does not belong to the set of the maximisers of the linear program, it is true that

$$\pi_H(X_H - \hat{R}_H(W)) + (W - \hat{S}_H(W))(1 + r) < \pi_H(X_H - \tilde{R}_H(W)) + (W - \tilde{S}_H(W))(1 + r)$$

or

$$\pi_H \hat{R}_H(W) - (I - \hat{S}_H(W))(1 + r) > \pi_H \tilde{R}_H(W) + (I - \tilde{S}_H(W))(1 + r) \geq 0$$

where the last inequality follows from constraint (1). Consider contract $\psi^d = (S^d, R^d)$, where $S^d = \epsilon \hat{S}_H + (1 - \epsilon)\tilde{S}_H$ and $S^d = \epsilon \hat{R}_H + (1 - \epsilon)\tilde{R}_H$. Constraints (1)-(3) are all satisfied, as contract ψ^d is a convex combination of contracts $\tilde{\psi}_H$ and $\hat{\psi}_H$ that both satisfy Constraints (1)-(3) (recall also Lemma 1). Furthermore, the payoff of type H with wealth W from contract ψ^d is

$$\epsilon[\pi_H(X_H - \hat{R}_H(W)) + (W - \hat{S}_H(W))(1+r)] + (1-\epsilon)[\pi_H(X_H - \tilde{R}_H(W) + (W - \tilde{S}_H(W))(1+r)]$$

Because this is strictly greater than $\pi_H(X_H - \hat{R}_H(W)) + (W - \hat{S}_H(W))(1 + r)$ for every $\epsilon \in (0, 1)$ and $\pi_H R^d - (I - S^d)(1 + r) > 0$, there is a contradiction with Condition (ii) of Definition 1.

Suppose now that $\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset$. For $\tilde{\psi}_H(W) = (\tilde{S}_H(W), \tilde{R}_H(W))$, Constraint (1) should be binding; otherwise, there is an immediate contradiction with Condition (ii) of Definition 1. Suppose then that Constraint (1) is binding. If Constraint (1) is binding, Constraint (3) is slack. Consider contract $\psi^\varepsilon = (\tilde{S}_H(W), \tilde{R}_H(W) + \varepsilon)$, where

$$\frac{I}{\pi_H} - \frac{\tilde{S}_H(W)(1+r)}{\pi_H} - \tilde{R}_H(W) < \varepsilon < X_H - \frac{\tilde{S}_H(W)(1+r)}{\pi_H} - \tilde{R}_H(W)$$

Because contract ψ^ε decreases the payoff of type L, Constraint (2) is satisfied. Moreover, $\pi_H(\tilde{R}_H(W) + \varepsilon) - (I - \tilde{S}_H(W))(1+r) > 0$, which contradicts Condition (ii) of Definition 1. Q.E.D.

□ **Proof of Proposition 3:** Due to Proposition 2, to characterise $\hat{\alpha}_H(W|\hat{\mu})$ for every W, it suffices to solve the linear program (recall also Proposition 1). Suppose that this has a solution (i.e., the set of maximisers is not the empty set) and this is denoted by $\hat{\psi}_H(W) = (\hat{S}_H(W), \hat{R}_H(W))$. It is only straightforward to show that at least one of Constraints (1)-(3) is binding. Suppose first that Constraint (1) is binding. Solving for $\hat{R}_H(W)$, one obtains

$$\hat{R}_H(W) = \frac{(I - \hat{S}_H(W))(1+r)}{\pi_H}$$

Constraint (2) is satisfied if and only if

$$\hat{S}_H \geq \overline{\overline{W}}(r) \equiv \overline{\overline{W}}(r) \equiv \frac{X_L - \frac{I(1+r)}{\pi_H}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H}\right)(1+r)} \tag{A.2}$$

The payoff of a type-H entrepreneur with wealth W is $\pi_H X_H - I(1+r) + W(1+r)$. Therefore, contract $(\hat{S}_H, \hat{R}_H)$ satisfies Constraint (3).

Suppose instead that Constraint (2) is binding. Solving for $\hat{R}_H(W)$, one obtains

$$\hat{R}_H(W) = X_L - \frac{\hat{S}_H(W)(1+r)}{\pi_L} \tag{A.3}$$

This is strictly decreasing in $\hat{S}_H(W)$, and hence, $(\hat{S}_H(W), \hat{R}_H(W))$, where $\hat{R}_H(W)$ is as specified in Eq. (A.2), is a solution to the linear program only if $\hat{S}_H(W) = W$. Constraint (3) is satisfied if and only if

$$W \geq \overline{W}(r) \equiv \frac{\Delta X}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H}\right)(1+r)} \tag{A.4}$$

One can further verify that Constraint (1) is satisfied if and only if $\overline{W}(r) \leq W \leq \overline{\overline{W}}(r)$. Assumption 1 ensures that $\overline{W}(r) < \overline{\overline{W}}(r)$. Therefore, if (A.3) is not satisfied, the set defined by Constraints (1)-(3) is the empty set. Hence, $\hat{\alpha}_H(W|\hat{\mu}) \notin \emptyset$ if and only if (4) is satisfied. Q.E.D.

□ **Proof of Proposition 6:** Due to Proposition 5, to characterise $\hat{\alpha}_H(W|\hat{\mu})$ for every W, it suffices to solve the linear program (recall also Proposition 4). Suppose that this has a solution (i.e., the set of maximisers is not the empty set) and this is denoted by $(\hat{\psi}_H(W), \hat{j}(W)) = ((\hat{S}_H(W), \hat{R}_H(W)), \hat{j}(W))$. It is only straightforward to show that at least one of Constraints (5)-(7) is binding.

First, suppose that $\hat{j}(W) = b$ and Constraint (5) is binding. Constraint (6) is satisfied if and only if

$$\hat{S}_H(W) \geq \frac{X_L - \frac{I(1+r)}{\pi_H^b}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H^b}\right)(1+r)}$$

Constraint (7) is satisfied if and only if $X_H \geq \frac{I(1+r)}{\pi_H^b}$.

Suppose instead that Constraint (6) is binding. Solving for $\hat{R}_H(W)$, one obtains

$$\hat{R}_H(W) = X_L - \frac{\hat{S}_H(W)(1+r)}{\pi_L} \tag{A.5}$$

This is strictly decreasing in $\hat{S}_H(W)$, and hence, $\hat{\psi}_H(W) = (\hat{S}_H(W), \hat{R}_H(W))$, where $\hat{R}_H(W)$ is as specified in Eq. (A.5), is a solution to the linear program only if $\hat{S}_H(W) = W$. Constraint (7) is satisfied if and only if

$$W \geq W_1(r) \equiv \frac{\Delta X}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H^b}\right)(1+r)} \tag{A.6}$$

where

$$\frac{X_L - \frac{I(1+r)}{\pi_H^b}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H^b}\right)(1+r)} \geq W_1(r)$$

if and only if $X_H \geq \frac{I(1+r)}{\pi_H^b}$. This implies that if $X_H < \frac{I(1+r)}{\pi_H^b}$, undertaking the baseline project is never part of an equilibrium.

Now, suppose that $\hat{j}(W) = a$ and Constraint (5) is binding. Constraint (6) is satisfied if and only if

$$\hat{S}_H(W) \geq \frac{X_L - \frac{I(1+r)}{\pi_H^a}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H^a}\right)(1+r)} \tag{A.7}$$

whereas it is true that

$$\pi_H^a(X_H - \hat{R}_H) + (W - \hat{S}_H(W))(1+r) - c \geq \pi_H^b(X_H - \hat{R}_H(W)) + (W - \hat{S}_H(W))(1+r) \tag{A.8}$$

(i.e., type H with wealth W has an incentive to undertake the advanced project in lieu of the baseline project) if and only if

$$\hat{S}_H(W) \geq W_3(r) \equiv I - \frac{\pi_H^a}{1+r}\left(X_H - \frac{c}{\Delta\pi_H}\right) \tag{A.9}$$

If Constraint (6) is binding, then Eq. (A.8) is satisfied if and only if

$$\hat{S}_H(W) \geq W_2(r) \equiv \frac{\Delta X + \frac{c}{\Delta\pi_H}}{\frac{1}{\pi_L}(1+r)} \tag{A.10}$$

and Constraint (5) is satisfied if and only if Eq. (A.7) is not satisfied.

If $X_L > \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$ (i.e., the hypothesis of the proposition) and $X_L < X_H + \left(\frac{\pi_H^b}{\pi_L} - 1\right)\frac{c}{\Delta\pi_H}$ (i.e., Assumption 2), then

$$W_3(r) < W_2(r) < \frac{X_L - \frac{I(1+r)}{\pi_H^a}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H^a}\right)(1+r)}$$

and

$$W_1(r) < W_2(r) < \frac{X_L - \frac{I(1+r)}{\pi_H^a}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H^a}\right)(1+r)}$$

Therefore, if $X_H \geq \frac{I(1+r)}{\pi_H^b}$, then

$$\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset, \text{ for every } W < W_1(r) \equiv \frac{\Delta X}{(\frac{1}{\pi_L} - \frac{1}{\pi_H^b})(1+r)} \tag{i}$$

$$\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,b) : m \in \hat{\mu}(W)\}, \text{ for every } W_1(r) \leq W \leq W_2(r) \equiv \frac{\Delta X + \frac{c}{\Delta\pi_H}}{\frac{1+r}{\pi_L}} \tag{ii}$$

$$\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,a) : m \in \hat{\mu}(W)\}, \text{ for every } W_2(r) \leq W \leq +\infty \tag{iii}$$

whereas if $X_H < \frac{I(1+r)}{\pi_H^b}$, then

$$\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset, \text{ for every } W < W_2(r) \tag{i}$$

$$\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,a) : m \in \hat{\mu}(W)\}, \text{ for every } W_2(r) \leq W \leq +\infty \tag{ii}$$

□ **Proof of Proposition 7:** Let $W_1(r)$, $W_2(r)$ and $W_3(r)$ be defined as in Eqs. (A.6), (A.10) and (A.9), respectively. If $X_L \leq \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$ (i.e., the hypothesis of the proposition) and $X_L < X_H + \left(\frac{\pi_H^b}{\pi_L} - 1\right)\frac{c}{\Delta\pi_H}$ (i.e., Assumption 2)

$$\frac{X_L - \frac{I(1+r)}{\pi_H^a}}{\left(\frac{1}{\pi_L} - \frac{1}{\pi_H^a}\right)(1+r)} \leq W_2(r) \leq W_3(r)$$

and

$$W_1(r) < W_2(r) \leq W_3(r)$$

with strict inequalities if and only if $X_L < \frac{I(1+r)}{\pi_L} - \left(\frac{\pi_H^a}{\pi_L} - 1\right)\left(X_H - \frac{c}{\Delta\pi_H}\right)$ Therefore, if $X_H \geq \frac{I(1+r)}{\pi_H^b}$, then

$$\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset, \text{ for every } W < W_1(r) \tag{i}$$

$$\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,b) : m \in \hat{\mu}(W)\}, \text{ for every } W_1(r) \leq W \leq W_3(r) \equiv I - \frac{\pi_H^a}{1+r}\Big(X_H - \frac{c}{\Delta\pi_H}\Big) \tag{ii}$$

$$\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,a) : m \in \hat{\mu}(W)\}, \text{ for every } W_3(r) \leq W \leq +\infty \tag{iii}$$

whereas if $X_H < \frac{I(1+r)}{\pi_H^b}$,

$$\hat{\alpha}_H(W|\hat{\mu}) \in \emptyset, \text{ for every } W < W_3(r) \tag{i}$$

$$\hat{\alpha}_H(W|\hat{\mu}) \in \{(m,a) : m \in \hat{\mu}(W)\}, \text{ for every } W_3(r) \leq W \leq +\infty \tag{ii}$$

□ **Proof of Corollary 1:** From Propositions 6 and 7, aggregate investment for the four cases is given by

$$AI(r) = \underbrace{\lambda_H I\Big[F(W_2(r)) - F(W_1(r))\Big]}_{I^b(r)} + \underbrace{\lambda_H I\Big[1 - F(W_2(r))\Big]}_{I^a(r)} \tag{i}$$

$$AI(r) = \underbrace{\lambda_H I\Big[1 - F(W_2(r))\Big]}_{I^a(r)} \tag{ii}$$

$$AI(r) = \underbrace{\lambda_H I\Big[F(W_3(r)) - F(W_1(r))\Big]}_{I^b(r)} + \underbrace{\lambda_H I\Big[1 - F(W_3(r))\Big]}_{I^a(r)} \tag{iii}$$

$$AI(r) = \underbrace{\lambda_H I\Big[1 - F(W_3(r))\Big]}_{I^a(r)} \tag{iv}$$

Because $\partial W_1(r)/\partial r < 0$, $\partial W_2(r)/\partial r < 0$ and $\partial W_3(r)/\partial r > 0$, the derivatives as stated in Corollary 1 are established.

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