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POLLUTION ACCUMULATION AND ABATEMENT POLICIES IN TWO SUPPLY CHAINS UNDER VERTICAL AND HORIZONTAL COMPETITION AND STRATEGY TYPES

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Abstract. This research investigates the impact of both horizontal and vertical competition, on the one hand, and strategy types (commitment-based *versus* contingent-based equilibrium strategies), on the other hand, on the pollution accumulated by two supply chains over time. We consider a two-stage game model where two manufacturers and two retailers are involved in a wholesale price contract, in order to supply the demand over a finite time horizon. In the first stage of the game, the manufacturers set their respective optimal transfer prices. During the second stage, polluting emissions are created over time in proportion with demand, which is controlled by the retailers' respective consumer prices. In this stage, the manufacturers are involved in emissions abatement. In this setup, we seek to identify the combination of market structure and strategy type that leads the two supply chains to generate the lowest pollution intensity and the highest level of abatement intensity.

Keywords. Pollution accumulation, Price competition, Double marginalization, Supply chains.

1. Introduction

Although much has been written about the importance of environmental performance (e.g., Chava, 2014; Flammer, 2015) and the impact of environmental regulation on firms' competitiveness (Porter, 1991; Porter and van der Linde, 1995; Xepapadeas and de Zeeuw, 1999), the influence of competition on pollution accumulation has been neglected. Competition has two dimensions: horizontal and vertical. Horizontal competition is usually based on product substitutability that leads producers to compete either on price or on quantity (e.g., Singh and Vives, 1984). In contrast, a usual feature of vertical competition between manufacturers and retailers is the well-known double marginalization (Spengler, 1950), which results in higher consumer prices, lower demand, and smaller profits than if decisions were centralized.

The environmental economics literature has dealt with the consequences of a lack of horizontal internalization of pollution externalities in a decentralized setting where non-competing producers draw utility from polluting emissions as a by-product of production and incur a cost related to the pollution stock (for a survey, see, e.g., Jørgensen *et al.*, 2010; Benchekroun and Long, 2011). One serious outcome of this setting is that strategy types, i.e., whether producers' strategy is precommitted to emissions reduction or contingent upon the pollution stock do matter. Along with strategy types, both horizontal and vertical competition can affect pollution accumulation.

Consider the apparel industry where the polluting emissions have been serious concerns. While many fast fashion companies such as H&M and Gap use outsourcing extensively without keeping in-house capacity, some retailers such as Zara and Uniqlo maintain a vertically integrated chain where the design and the production also take place in-house. In the oil industry, major companies such as Shell, Chevron and BP manage from upstream exploration to marketing and sales of refined products, whereas non-integrated super-independents as Anadarko, Apache or Talisman Energy consists in more disintegrated, i.e., competing, supply chains (Boscheck, 2006). Mergers and acquisitions also

take place actively in this industry, e.g., Chevron's recent acquisition of Anadarko in a \$33 billion deal (Casey *et al.*, 2019), implying the tendency toward horizontal integration.

Distinct patterns of competition between firms, vertically and horizontally, influence abatement and pricing decisions and the pollution of firms in industries, and several studies have explored this aspect based on the static approach as a first step. However, an important element that has been largely ignored is the intertemporal dimension of the pollution externalities. For instance, pollutants are generally long-lasting and cumulative in nature (Arrow *et al.*, 1995) and firms typically pay attention to the pollution from the long-term perspective e.g., BP planning to keep its emissions around 50 million tonnes of CO₂ by 2025 and Shell setting emission targets for 2030. This study directly incorporates the intertemporal feature of the pollution externalities based on dynamic games in investigating competition and pollution accumulation, thus filling the important missing element in the literature and also responding to the call for studies on dynamic settings with regard to environmental problems (e.g., Xu *et al.*, 2017; Galbreth and Ghosh, 2013).

Another important observation that motivates our research relates to strategy types. While contingent strategies are common in practice, a growing number of global producers (e.g., Ikea, Coca-Cola, Dell, General Mills, Sony,...) recently decided to opt for commitment strategies (<https://sciencebasedtargets.org/companies-taking-action>). Whether to adopt commitment or contingent strategy is of particular concern for many firms these days along with a global movement to tackle climate change. For instance, Shell, which previously hesitated to employ any pre-committed strategies, recently made an important decision to precommit to their strategies for three to five years to reduce emissions, by 20 percent by 2035 (Raval *et al.*, 2018; van Beurden, 2018). BP also pledged to invest \$100 million over the next three years for emission reduction in exploration and production (Herron, 2019). Top apparel companies also recently adopted commitment strategies to emission reductions, including major brands such as Gap, Nike, Levi's, Guess, H&M, Zara and Puma (WRI, 2017). Similar examples abound across a wide variety of industries, e.g., Siemens precommitted to invest \$110 million over the three years specifying the details on how the investment in emission reduction will be realized (Kaeser, 2015), and Procter and Gamble committed to their set of actions to cut its emissions in half (P&G, 2018).

Which strategy types (commitment versus contingent) firms adopt is not only of practical importance, but also a critical factor in studying competition and pollution. Since firms under competition interact with each other through their strategic choices, the way firms make their decisions can completely alter the competitive landscape between firms. In this regard, whether firms precommit to their course of actions at the beginning, or revise them continuously throughout the competitive interaction have been regarded as an important theoretical distinction in terms of behaviors of competing firms (Kamien and Schwartz, 2012; Dockner *et al.*, 2000). These two strategies make contrasting assumptions on firms' behaviors, and strategic interactions of competing firms are usually more pronounced under contingent strategy compared to under commitment strategy. Therefore, we

examine how the strategy type are affected by competition and integration in an industry to determine the pollution accumulation and abatement policies of firms.

The following research questions are addressed:

- How are pollution accumulation and abatement related to the presence of competition and integration in an industry? That is, how do horizontal and vertical internalization of pollution externalities affect abatement policies and pollution accumulation in a bilateral duopoly?
- How do strategy types modify the relative impact of horizontal and vertical competition on pollution accumulation, if any? More specifically, how does the lack of precommitment by firms influence the pollution intensity, abatement intensity and firms' profits?

We briefly summarize the key results as follows:

- First, horizontal competition leads to more pollution than horizontal integration. The rationale behind this is twofold: a 'horizontally-induced volume effect' due to more competitive sales prices, leading to greater sales hence more pollution, on the one hand, and a 'horizontally-induced polluting effect' resulting from lower incentives for abatement efforts due to competition, making the manufacturing process dirtier.
- Second, vertical competition results in lower pollution than vertical integration. This is because double marginalization under vertical competition reduces the gross emissions via sales, on the one hand, and induces relatively greater manufacturers' abatement effort, on the other hand. These two effects are defined as 'vertically-induced volume effect' and 'vertically-induced polluting effect'.
- Third, the vertically-induced volume effect is greater than the horizontally-induced volume effect and the horizontally-induced polluting effect is greater than the vertically-induced polluting effect. That is, the volume effect depends more on the vertical configuration, whereas the polluting effect depends more on the horizontal configuration.
- Finally, the impact of a strategy type depends on market configurations. Under horizontal competition, a commitment strategy leads to mitigating the polluting effect compared to a contingent strategy. The impact of a strategy type on the volume effect depends on the market structure. Under horizontal competition, strategy types influence abatement and pricing paths of competing firms and a commitment strategy leads to higher profitability for supply chain entities. In the context of horizontal integration, both strategy types coincide.

The paper proceeds as follows. Section 2 reviews the relevant literature. In section 3, we develop a dynamic game model with two manufacturers and two retailers. Section 4 successively derives the optimal policies. Section 5 compares the strategies and their outcomes both analytically and numerically. Section 6 concludes the paper.

2. Literature review

This study is related to two streams of literature on pollution accumulation: competition and strategy types. We first review the studies on the impacts of horizontal and vertical competition on the

pollution, and then review the literature on the strategy type, i.e., pre-commitment and contingent strategies, and its impact on pollution accumulation.

2.1. Horizontal and vertical competition and pollution

2.1.1. Horizontal competition and pollution.

Many studies examine the impact of regulation or consumer awareness under horizontally competitive market in the environmental context (e.g., Ouchida and Goto, 2016; Subramanian *et al.* 2007). Sabzevar *et al.* (2017) investigate the effects of cap-and-trade policy, and analyze whether tighter policies increase the competitive firms' profits. Galbreth and Ghosh (2013) examine the impact of environmental awareness on two horizontally competing firms with different sustainability levels. They find that higher awareness may benefit not only the more sustainable firm, but also the less sustainable one. The impacts of different power structures are also often examined under horizontal competition. Meng *et al.* (2018) show that firms' optimal product strategies between common and low-carbon products depend on the power structure, i.e., Nash versus Stackelberg structures. Chen *et al.* (2017) also examine how the market power between two competing manufacturers influence the pricing and emissions reduction strategies.

How horizontal competition influences the strategic interaction between firms and the environment is gaining increasing attention, wherein the advantage of horizontal competition has been largely supported. Yalabik and Fairchild (2011) show that if the degree of environmental competition between firms is high, it encourages more environmental innovation of competing firms. Yenipazarli (2019) finds that a firm under duopoly competition exerts more abatement efforts than a monopolist firm to differentiate itself from its competitor when the abatement requires a relatively large up-front investment. Ouchida and Goto (2016) further show that even when competing firms can coordinate emission-reducing R&D to maximize joint profits, such cooperation can be socially inferior in cases of severe environmental damage and low investment costs. Luo *et al.* (2016) investigate the competing manufacturers' pricing and abatement strategies under the cap-and-trade policy and argue that while horizontal competition increases emissions, it leads to higher green technology investments. Contrary to the aforementioned papers based on static games, we find that horizontal competition lessens the abatement incentive of competing firms because the intertemporal pollution externalities are not internalized by the competitive manufacturers. We also find that strategy types influences the degree to which horizontal competition reduces abatement incentive of firms.

2.1.2. Vertical competition and pollution.

Due to the well-known double marginalization phenomenon, a rich body of the literature on vertical competition explores how the coordinating contracts in the decentralized chain can achieve the system efficiency in the centralized chain (see Tsay *et al.* 1999 for a review). As a direct extension, many environmental studies on vertical competition explore how diverse contracts may influence the environmental performance (e.g., Zhou et al., 2016; Du *et al.*, 2015). Ghosh and Shah (2015) show

that while the integrated chain leads to the highest product greening levels, the greening level in the competitive chain becomes also higher with the cost sharing than without. Yang and Chen (2018) show that under consumer awareness and carbon tax, sharing both of revenue and abatement investment cost results in the same equilibrium outcome as sharing revenue only, thus cost sharing contract becomes dispensable. Hong and Guo (2018) examine price-only, green-marketing cost-sharing, and two-part tariff contracts in a supply chain and show that increased cooperation may not be beneficial to all entities in a supply chain, but it can lead to the highest product greenness. Some environmental studies consider bargaining power between manufacturers and retailers in the vertically competitive chain (e.g., Ni *et al.* 2010; Ghosh and Shah, 2012). Wang *et al.* (2016) examine the emissions reduction under the retailer dominant and the power balanced cases, with the vertically integrated chain as a benchmark.

The underlying theme of the aforementioned studies is the inefficiency of vertical competition in the environment context. However, recent studies suggest that this belief may not always hold and vertical competition may not be detrimental, leading to the same or even better environmental sustainability. Zhu and He (2017) investigate the effects of supply chain structures, product types and competition type (price or greenness). They suggest that although the lack of vertical coordination results in less sales and less profit, it may not affect the optimal degree of product greenness. Xu *et al.* (2017) similarly show that the abatement level remains the same regardless of whether a manufacturer and a retailer coordinate their decisions, despite a decrease in production quantities and profits in a decentralized chain. Jaber *et al.* (2013) find that vertical coordination can reduce the total supply chain cost, but it can increase emissions and penalty costs. Chen *et al.* (2019) show that the vertically coordinated chain may result in higher emissions than the competitive chain when firms make order replenishment decisions. Toptal and Cetinkaya (2017) also find that, depending on production and inventory parameters, carbon emissions can be larger under a vertically coordinated system.

Our study extends this line of research that suggests the adverse environmental impact of vertical integration. However, there are important differences between our paper and the afore-cited studies in that Jaber *et al.* (2013), Chen *et al.* (2019), Toptal and Cetinkaya (2017) and Zhu and He (2017) do not consider pollution abatement and often assume a fixed demand, whereas we explicitly consider firms' investment in pollution abatement and assume an endogenously determined demand. Our study also differs from Xu *et al.* (2017) in that while they focus on coordinating contracts, we focus on strategy types that govern firms' behaviors over time. Furthermore, none of those studies consider intertemporal pollution externalities or the strategy types of firms.

2.1.3. Overall contribution to the literature on competition and pollution

Based on the literature review in the above, we summarize how this study contributes to the studies on the competition (horizontal and vertical) and the environment. First, despite the growing interests on the impacts of competition and integration on pollution, theoretical lenses that provide systematic

understanding of the results are largely lacking. To the best of our knowledge, our study is one of the first to identify and compare drivers of the impacts of competition on pollution accumulation, i.e., ‘volume effects’ and ‘polluting effects’, both vertically- and horizontally-induced.

Second, we find that strategy type may influence the effects of market structures, thus filling the gap in the literature where these interdependences are largely ignored. For instance, strategy type affects the degree of the polluting effect and the volume effect underlying the impact of competition on pollution accumulation and also the abatement and pricing trajectories of firms under distinct market structures.

Third, our study extends the literature in that we jointly investigate two dimensions of competition, i.e., vertical competition or integration, horizontal competition or integration. This setting allows to examine how the respective impacts interact in the markets where both types of competition exist, which is common in practice. A few recent studies consider both dimensions of competition along with environmental issues. Yang *et al.* (2017) suggest that vertical cooperation leads to higher abatement rate and upstream horizontal coordination may reduce downstream profits, from which the revenue-sharing contract can bring economic and environmental benefits. Creti and Sanin (2017) find that horizontal integration increases profits in the vertically-related market due to the decrease in emission permit price. Madani and Rasti-Barzoki (2017) show that horizontal cooperation under vertically competitive chains, where one chain selling green product and the other selling non-green product, leads to both eco-friendlier product and higher profits. Our study differs from these studies in two critical aspects: (i) Although these studies consider two dimensions of competition, the interactive impacts between vertical and horizontal structures remain mostly unexplored. In contrast, we show that vertical structure can significantly alter the effects of horizontal structure, and vice versa. (ii) While these papers adopt static games model, we employ dynamic games to incorporate the intertemporal feature of pollution externalities and investigate strategy types of competing firms, which determine the competitive landscape of firms.

2.2. Strategy types

While the majority of environmental studies are based on static models, our work is based on a dynamic game model and shows how firms’ emissions abatement, production and pricing behaviors would change over time. This approach is particularly useful in investigating dynamics of pollution accumulation, instead of one-shot emission (Jørgensen *et al.*, 2010). In this regard, we extend the bilateral monopoly game model in El Ouardighi *et al.* (2016) to a bilateral duopoly structure to assess the relative effects of price competition and double marginalization. This study differs from El Ouardighi *et al.* (2016) in that we characterize the main drivers of pollution accumulation, i.e., volume effects and polluting effects, and investigate their relative importance under horizontal and vertical market configurations, which shows that while volume effects are influenced more by vertical configurations, polluting effects are influenced more by horizontal configurations. We also investigate

the interaction between competition and strategy types and find that the effects of strategy types can be significant depending on market configurations.

In the dynamic game models for firms' emissions, the impact of strategy types has been investigated, especially for a commitment strategy (i.e., an open-loop Nash equilibrium; OLNE) and for a contingent strategy (i.e., a feedback Nash equilibrium; FBNE). Contingent strategy has been generally considered to generate lower public goods since it often leads to free-riding behaviors between players, which results in higher pollution accumulation than commitment strategy (see Dockner *et al.* 2000; El Ouardighi *et al.*, 2018, b). Hoel (1993) also suggest that contingent strategies result in higher total emissions, however, they reveal that the tax giving a Pareto optimal can be the same for both strategies. El Ouardighi *et al.* (2016) further show that the two strategies may coincide due to the linear state property involved. Bertinelli *et al.* (2014) also find that the contingent strategies may even lead to lower long-run carbon emissions if the pollution is initially high. Feenstra *et al.* (2001) find that the strategy type can alter the effectiveness of tax versus standard: although standards are better under commitment strategy, tax can be better due to the increased strategic interaction if firms adopt contingent strategies. Stimming (1999) shows that under duopolistic competition, stricter policy toward one firm may result in higher total emissions due to the higher investment of its competitor and contingent strategies may strengthen this effect.

We fill the gap in this literature in two major aspects. First, although the aforementioned studies explore different strategy types in the context of pollution accumulation, the impacts of market configurations are largely overlooked. To be specific, most studies consider horizontal competition as a given context, while neglecting vertical competition, which is another important dimension of strategic interaction between firms. Zhang *et al.* (2016) examine vertical competition for both strategy types and show that the level of energy efficiency is higher under contingent strategies. However, they focus on demand evolution and energy efficiency, without considering any pollution-specific dynamics. Zu *et al.* (2018) and Zhou and Ye (2018) consider vertical competition with environmental regulation for an infinite horizon, but they do not investigate the effects of different strategy types.

Second, although previous studies reported varying results on the effects of strategy types, the driving forces remain largely unclear. We add to the literature by showing that the underlying effect of strategy types can further depend on market structures. For instance, while contingent strategies lead to more pollution than commitment strategies due to lower abatement under TC, the environmental demerit is strengthened under HCVI due to both lower abatement and greater gross emissions.

3. Model formulation

To answer the research questions, we consider a bilateral duopoly with two manufacturers and two retailers (Figure 1). Manufacturer i sells a finished good to retailer i , $i = 1, 2$, which resells it, meeting a consumer demand. The retailers compete on price for the demand. The manufacturers generate

polluting emissions and incur a cost related to their accumulation. To reduce the pollution stock, the manufacturers have the opportunity to invest in abatement efforts.

In this study, we employ the dynamic games approach that allows much deeper understanding of the pollution problem than static games. Specifically, dynamic games capture the intertemporal dimension of pollution externalities, which cannot be dealt with in static games. This intertemporal feature implies that firms' strategic decisions and pollution essentially evolve together over time, both impacting each other: on the one hand, decisions of competing firms influence pollution accumulation, but on the other hand, competing firms' decisions are also shaped by the pollution stock based upon which they can condition their profit-maximizing abatement, pricing and production paths over time. Dynamic games are of considerable value in dealing with these intertemporal dynamics underlying pollution externalities. In our study, another important advantage of dynamic games approach is that it provides mathematically distinctive equilibrium concepts related to strategy types (commitment, contingent), which govern strategic interaction between firms competing over time. In this regard, dynamic games facilitate a more rigorous analysis of our key research questions.

Most often, transactions between a manufacturer and a retailer are governed by a conventional wholesale price contract (WPC), under which the transfer price for each unit ordered by the retailer is set to maximize the manufacturer's profit. Under a WPC, double marginalization is common. To study the role of double marginalization, we consider a two-stage game model where each manufacturer and one retailer first agree on a WPC, and then supply the demand over a finite time horizon. At the first stage of the game, the manufacturers set their respective optimal transfer prices. During the second stage, polluting emissions are generated over time in proportion with demands, which are in turn controlled by the competing retailers' respective consumer prices. In this stage, the manufacturers are involved in emissions abatement.

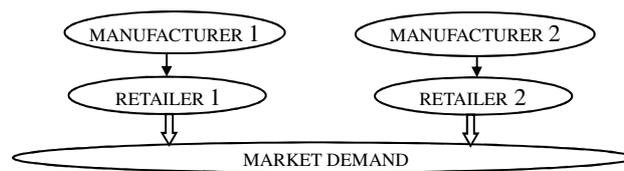


Fig. 1. Price-competing supply chains

We seek to analyze how pollution accumulation and abatement are related to the presence of competition and integration in an industry. Based on competition or integration at vertical and horizontal levels, four market structures are considered:

- A monopoly market, where the manufacturers and retailers are all centralized to maximize their joint profits (total integration);
- A bilateral duopoly market, where the manufacturers and retailers are all decentralized to maximize their individual profits (total competition);
- A bilateral monopoly market, where integrated manufacturers compete with integrated retailers to maximize their horizontal profits (horizontal integration and vertical competition);

- A duopoly market, where integrated supply chains compete with each other to maximize their vertical profits (horizontal competition and vertical integration).

In our model, integration typically corresponds to perfect coordination between firms, while competition reflects a lack of coordination. Except in the total integration case, the firms can select either a commitment strategy (OLNE) or a contingent strategy (FBNE):

- Firms' strategies depend on time only, that is, each firm commits to his entire course of action at the outset, and the information on pollution stock is disregarded by (or unavailable to) firms (OLNE).
- Firms condition their actions both on the current value of this stock and on time (FBNE).

In addition, we assume a make-to-order system where no firm carries an inventory. Each manufacturer controls its transfer price to its retailer, which in turn determines its consumer price. Retailers are in competition for the demand whose substitution effects are based on consumer price.

We assume price-dependent demands with imperfect product substitutability. Function $D_i(p_i(t), p_j(t)) \geq 0$, $i = 1, 2, i \neq j$, represents retailer i 's demand, which is determined by price competition such that it decreases in retailer i 's price and increases in the rival's price. Therefore, price competition enables a retailer to improve its demand at its rival's expense. Demand $D_i(t)$ is specified as a linear function, that is:

$$D_i(t) = \alpha_i - \beta_i p_i(t) + \gamma_i p_j(t) \quad (1)$$

where $\alpha_i > 0$, denotes retailer i 's potential market, and $0 \leq \gamma_i < \beta_i$ are parameters, $i = 1, 2, i \neq j$, such that β_i denotes retailer i 's demand sensitivity to its own price, and γ_i reflects the degree of product substitutability between retailer i and its rival, $i = 1, 2$ (e.g., Ha *et al.*, 2011; Wu *et al.*, 2009). Note that $\gamma_i = 0$, $i = 1, 2$, corresponds to the case where the retailers are not in direct competition. Accordingly, the demands are non-negative if $p_i(t) \leq (\alpha_i + \gamma_i p_j(t)) / \beta_i$, $i = 1, 2, i \neq j$.

Assuming that each unit of a good produced and sold results in ψ_i units of emitted pollutant, the dynamics of the pollution stock, $y(t) \geq 0$, are given by:

$$\dot{y}(t) = \sum_{i \neq j} \psi_i (\alpha_i - \beta_i p_i(t) + \gamma_i p_j(t)) - \sum_i \varphi_i v_i(t), \quad y(0) = 0 \quad (2)$$

where $v_i(t) \geq 0$ is manufacturer i 's abatement effort, and $\varphi_i > 0$ is the marginal effectiveness of manufacturer i 's abatement effort, $i = 1, 2$.

In equation (2), the total current emissions result from the sum of the demands, and are therefore influenced both by the horizontal interaction between the retailers (consumer price competition) and by the vertical interaction between manufacturers and retailers (double marginalization). Further, manufacturer i 's abatement efforts are independent of the corresponding emissions, $\psi_i (\alpha_i - \beta_i p_i(t) + \gamma_i p_j(t))$, $i = 1, 2$. In this regard, the constraints $v_i(t) \leq \psi_i (\alpha_i - \beta_i p_i(t) + \gamma_i p_j(t)) / \varphi_i$, $i = 1, 2, i \neq j$, should be satisfied so that manufacturer i 's net current emissions remain non-negative, which implies that $\dot{y}(t) \geq 0$, $\forall t \geq 0$.

To account for recent empirical observations that terrestrial and oceanic natural pollution sinks are tending to exhaustion and will eventually become pollution sources (Cox *et al.*, 2000; Zhao and Running, 2010), no natural abatement of the pollution stock is considered in (2), which is also consistent with the assumption of a finite time horizon. To account for the accountability of the bilateral duopoly in the pollution accumulation, the initial pollution stock in (2) is normalized to zero, i.e., $y(0) = 0$.

Given that each firm's objective is to maximize its cumulative profits over a finite time horizon $[0, T]$, the discounting of future profits is omitted. Manufacturer i 's instantaneous gross profit is given by $(\omega_i - c_i)(\alpha_i - \beta_i p_i(t) + \gamma_i p_j(t))$, where $\omega_i > 0$ is manufacturer i 's transfer price to retailer i , and $c_i \geq 0$ is manufacturer i 's unit production cost, $i = 1, 2, i \neq j$. Manufacturer i 's transfer price to retailer i , ω_i , $i = 1, 2$, is chosen to determine an optimal WPC that maximizes manufacturer i 's individual profit. The retailer can then accept or reject the contract.

In general, most of the polluting emissions are related to the manufacturing process (Krass *et al.*, 2013). In return, the accumulation of the polluting emissions causes environmental externalities (or damage), which reduce the flow of useful services generated by the natural system to the manufacturing system. That is, the pollution stock entails a cost that is incurred by the manufacturers only. The manufacturers can thus invest in abatement efforts to reduce the pollution stock and the related costs. The negative externalities of the pollution stock are valued as an increasing convex cost function of pollution, that is, $f_i y(t)^2/2$, where $f_i > 0$, is manufacturer i 's pollution cost coefficient, $i = 1, 2$. This quadratic form, which implies that manufacturer i 's cost would increase sharply when the pollution level becomes very high, is standard in the literature on pollution control (e.g., Kossioris *et al.*, 2008; Jørgensen *et al.*, 2010; El Ouardighi *et al.*, 2018, a; ; El Ouardighi *et al.*, 2018, b). Due to diminishing returns to scale, the manufacturers incur a quadratic cost function for the abatement efforts, that is, $e_i v_i(t)^2/2$ where $e_i > 0$, is manufacturer i 's abatement effort cost coefficient, $i = 1, 2$. A similar quadratic form is used in Liu *et al.* (2012) and Bertinelli *et al.* (2014).

In contrast, retailer i 's current net profit is the difference between its gross sales revenue and its procurement cost from the manufacturer, that is, $(p_i(t) - \omega_i)(\alpha_i - \beta_i p_i(t) + \gamma_i p_j(t))$, $i = 1, 2, i \neq j$. We assume equivalent parameter values for both retailers, that is, $\alpha_i \equiv \alpha > 0$, $\beta_i \equiv \beta > 0$, $\gamma_i \equiv \gamma \geq 0$, $i = 1, 2$, so that any differences in the way sales are affected by price competition do not stem from arbitrary decisions concerning parameter values. Also, for simplicity, we assume symmetric parameter values for manufacturer i 's abatement effort's marginal effectiveness, $\varphi_i \equiv \varphi > 0$, manufacturer i 's pollution cost coefficient, $f_i \equiv f > 0$, and the pollutant per unit production, $\psi_i \equiv \psi > 0$, $i = 1, 2$. Without loss of generality, manufacturer i 's unit production cost, c_i , and manufacturer i 's abatement cost coefficient, e_i , are respectively normalized as $c_i = 0$ and $e_i = 1$, $i = 1, 2$.

$y(t) \geq 0$	Pollution stock at t
$v_i(t) \geq 0$	Manufacturer i 's pollution abatement effort at t , $i = 1, 2$
$p_i(t) \geq 0$	Retailer i 's consumer price at t , $i = 1, 2$

$D_i(t) \geq 0$	Retailer i 's demand at t , $i = 1,2$
$\omega_i \geq 0$	Manufacturer i 's transfer price, $i = 1,2$
J^{m_i}, J^{r_i}	Manufacturer i 's cumulative profit, Retailer i 's cumulative profit, $i = 1,2$
J^m, J^r	Cumulative profit of the manufacturers, Cumulative profit of the retailers
J, J^i	Cumulative profit of the entire system, Cumulative profit of the supply chain i , $i = 1,2$
\bar{v}_i	Manufacturer i 's average pollution abatement effort ; i.e., $\bar{v}_i = \int_0^T v_i(t) dt/T$
ξ_i	Manufacturer i 's pollution abatement effort per unit of cumulative demand i.e., $\xi_i = \int_0^T v_i(t) dt / \int_0^T D_i(t) dt$
\bar{p}_i	Retailer i 's average consumer price, $i = 1,2$; i.e., $\bar{p}_i = \int_0^T p_i(t) dt/T$
D_{cum}	Retailer i 's cumulative demand, $i = 1,2$; i.e., $D_{cum} = \sum_i \int_0^T D_i(t) dt$
PI	The terminal pollution stock per unit of cumulative demand, i.e., $PI = y(T) / \sum_i \int_0^T D_i(t)$
AI	The total cumulative abatement effort per unit of terminal pollution stock, i.e., $AI = \sum_i \int_0^T v_i(t) / y(T)$
$\alpha_i > 0$	Retailer i 's potential market, $i = 1,2$;
$\beta_i > 0$	Marginal sensitivity of retailer i 's demand to own price, $i = 1,2$;
$\gamma_i > 0$	Marginal sensitivity of retailer i 's demand to retailer j 's price, $i, j = 1,2$, $i \neq j$
$\varphi_i > 0$	Marginal effectiveness of manufacturer i 's abatement effort, $i = 1,2$
$c_i \geq 0$	Manufacturer i 's unit production cost, $i = 1,2$
$e_i > 0$	Manufacturer i 's abatement effort cost coefficient, $i = 1,2$
$f_i > 0$	Manufacturer i 's pollution cost coefficient, $i = 1,2$
$\psi_i > 0$	Gross pollutant (before abatement) per unit production of manufacturer i , $i = 1,2$
$t \in [0, T]$	Time, $T < \infty$

Tab. 1. Main variables and parameters

Table 1 summarizes the main variables and parameters of the model. Note that we may use superscripts in notations to indicate the corresponding market configuration: TI , TC , $HIVC$, $HCVI$ denote total integration, total competition, horizontal integration and vertical competition, and horizontal competition and vertical integration, respectively. We may also use subscripts to distinguish the strategy type: ol and fb denote the OLNE strategy and the FBNE strategy, respectively.

	Vertical integration	Vertical competition
Horizontal integration	<u>Total integration</u> - Monopoly market structure - No double marginalization - Both vertical and horizontal internalization of pollution externalities - Price coordination between retailers	<u>Horizontal integration and Vertical competition</u> - Bilateral monopoly market structure - Double marginalization - Only horizontal internalization of pollution externalities - Price coordination between retailers
Horizontal competition	<u>Horizontal competition and Vertical integration</u> - Duopoly market structure - No double marginalization - Only vertical internalization of pollution externalities - Price competition between retailers	<u>Total competition</u> - Bilateral duopoly market structure - Double marginalization - Neither vertical nor horizontal internalization of pollution externalities - Price competition between retailers

Tab. 2. Main characteristics of the four configurations

The differential game model is then given as:

$$\text{Max}_{\omega_i, v_i \geq 0} J^{m_i} = \int_0^T \left[\omega_i (\alpha - \beta p_i + \gamma p_j) - \frac{v_i^2}{2} - \frac{fy^2}{2} \right] dt \quad (3)$$

$$\text{Max}_{p_i \geq 0} J^{r_i} = \int_0^T (p_i - \omega_i) (\alpha - \beta p_i + \gamma p_j) dt \quad (4)$$

subject to:

$$\dot{y}(t) = \sum_{i \neq j} \psi (\alpha - \beta p_i(t) + \gamma p_j(t)) - \varphi \sum_i v_i(t), \quad y(0) = 0 \quad (5)$$

$$p_i(t) \leq (\alpha + \gamma p_j(t)) / \beta, \quad v_i(t) \leq \psi (\alpha - \beta p_i(t) + \gamma p_j(t)) / \varphi$$

$i = 1, 2, i \neq j$. As will be shown below, the control constraints are never binding. We therefore do not explicitly derive the related optimality conditions. Four market configurations, whose main characteristics are described in Table 2, are analyzed.

4. Analysis

The next four subsections derive the pricing and abatement decisions in the contexts of monopoly, bilateral duopoly, bilateral monopoly and duopoly. Proofs of the following theorems and lemmas are provided in Appendix. Note also that cumulative profits under optimal strategies for all cases in subsections 4.1-4.4 are derived in Table A1 in Appendix.

4.1 Total integration

This subsection analyzes total integration, where the manufacturers and the retailers jointly centralize their decisions to maximize the joint profits, as in a monopoly. In this case, which corresponds to a one-stage game, there is perfect coordination between the firms both at the vertical level, because the marginalization is totally mitigated, and at the horizontal level, because the pollution externalities and the substitution effects are both vertically and horizontally internalized in the manufacturers' and retailers' decisions.

Theorem 1. *The equilibrium consumer prices and abatement efforts are unique and given by:*

$$v_i^{TI}(t) = \frac{\alpha\varphi\psi}{((\beta-\gamma)\psi^2+2\varphi^2)} \left[1 - \frac{\text{ch}(\theta t)}{\text{ch}(\theta T)} \right] \quad (6)$$

$$p_i^{TI}(t) = \frac{1}{2} \left\{ \frac{\alpha}{(\beta-\gamma)} + \frac{\alpha\psi^2}{((\beta-\gamma)\psi^2+2\varphi^2)} \left[1 - \frac{\text{ch}(\theta t)}{\text{ch}(\theta T)} \right] \right\} \quad (7)$$

$i = 1, 2$, where the superscript *TI* refers to "Total integration", and $\theta \equiv \sqrt{2f((\beta-\gamma)\psi^2+2\varphi^2)}$. Accordingly, the equilibrium pollution stock is unique and given by:

$$y^{TI}(t) = \frac{\alpha\psi\text{sh}(\theta t)}{\theta\text{ch}(\theta T)} \quad (8)$$

From (6) and (7), we observe that the greater the abatement efforts the higher the consumer prices. An intuitive explanation on the positive relationship is that manufacturers' and retailers' abatement and pricing decisions can be coordinated to reduce the pollution through greater abatement efforts and smaller volume, because pollution externalities are perfectly internalized in this case.

From (7), the retailer i 's consumer price is initially high and decreases concavely over time $t \in [0, T]$, which implies convexly increasing demands. From (6), the manufacturer i 's abatement effort is also initially high and concavely decreasing over time. These imply that firms under perfect coordination put more emphasis on mitigating the pollution accumulation during the initial periods, whereas they focus more on enlarging the demand volume in the later periods. Thus, the pollution stock is strictly and convexly increasing over time as shown in (8).

4.2 Total competition

This subsection deals with the total competition case, where the manufacturers and the retailers make

non-cooperative decisions to maximize their individual profits, as in a bilateral duopoly. In this case, there is no coordination between the players at the vertical level because double marginalization prevails between manufacturers and retailers, or at the horizontal level, because of the imperfect internalization of pollution externalities by the manufacturers and the price competition for demand between the retailers. In addition, the pollution stock is payoff-irrelevant to the retailers, which implies that pollution externalities are not at all internalized in the retailers' consumer prices.

Under total competition, (3)-(4)-(5) form a two-stage game whose sequence of events is as follows:

- In the first stage of the game, $t = 0$, manufacturer i sets the optimal transfer price for retailer i ;
- During the second stage, assuming that an agreement on the optimal WPC has been reached between manufacturer i and retailer i , $i = 1, 2$, each firm deploys its strategy over the time interval $[0, T]$.
- In the final period, $t = T$, the profits are allocated among the firms according to the WPC.

As usual in a two-stage game, one solves the game backward.

Theorem 2. *Under commitment strategies (OLNE), manufacturer i 's transfer price and abatement effort and retailer i 's consumer price are unique and given by:*

$$\omega_{iol}^{TC} = \frac{\{\beta(\beta-\gamma)\psi^2[(2-\text{sech}(\rho T)^2)\rho T - \text{th}(\rho T)] + 2\varphi^2\rho(2\beta-\gamma)T\} \alpha}{\{\beta(\beta-\gamma)\psi^2[(2-\text{sech}(\rho T)^2)\rho T - \text{th}(\rho T)] + 4\varphi^2\rho(2\beta-\gamma)T\} (\beta-\gamma)} \quad (9)$$

$$v_{iol}^{TC}(t) = \left[1 - \frac{\text{ch}(\rho t)}{\text{ch}(\rho T)}\right] \frac{2\alpha\beta\varphi\rho T\psi}{\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2) - \text{th}(\rho T)] + 4\varphi^2\rho(2\beta-\gamma)T} \quad (10)$$

$$p_{iol}^{TC}(t) = \frac{\alpha + \beta\omega_{iol}^{TC}}{2\beta-\gamma} \quad (11)$$

$i = 1, 2$, where the superscript TC refers to "total competition", and $\rho \equiv \varphi\sqrt{2f}$. Accordingly, the pollution stock is unique and given by:

$$y_{ol}^{TC}(t) = \frac{4\alpha\beta\varphi^2T\psi}{\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2) - \text{th}(\rho T)] + 4\varphi^2\rho(2\beta-\gamma)T} \frac{\text{sh}(\rho t)}{\text{ch}(\rho T)} \quad (12)$$

From (10) and (12), abatement efforts are initially high and concavely decrease to zero and the pollution stock is strictly increasing over time, similar to the total integration case. However, from (9) and (11), we observe that the transfer prices and the consumer prices under commitment strategies are both time-independent, which implies constant demands over the whole planning horizon. In particular, constant consumer prices result from the fact that pollution externalities are not internalized in the retailers' decision-making. Thus, retailers simply set their consumer prices to be proportional to the transfer price, considering the trade-off between their margin and demand, but without consideration of the externalities of their pricing decisions on the pollution.

Theorem 3. *Under contingent strategies (FBNE), manufacturer i 's transfer price and abatement effort and retailer i 's consumer price are:*

$$\omega_{ifb}^{TC} = \frac{\{[4\beta(\beta-\gamma)\psi^2 + 3\varphi^2(2\beta-\gamma)]\tau T - 4\beta(\beta-\gamma)\psi^2\text{th}(\tau T)\} \alpha}{\{[2\beta(\beta-\gamma)\psi^2 + 3\varphi^2(2\beta-\gamma)]\tau T - 2\beta(\beta-\gamma)\psi^2\text{th}(\tau T)\} 2(\beta-\gamma)} \quad (13)$$

$$v_{ifb}^{TC}(t) = \frac{\sqrt{f} \text{th}[\tau(T-t)]y(t)}{\sqrt{3}} + \frac{2\beta\psi[\alpha - (\beta-\gamma)\omega_{ifb}^{TC}]\{1 - \text{sech}[\tau(T-t)]\}}{3\varphi(2\beta-\gamma)} \quad (14)$$

$$p_{ifb}^{TC}(t) = \frac{\alpha + \beta \omega_{ifb}^{TC}}{2\beta - \gamma} \quad (15)$$

$i = 1, 2, \tau \equiv \varphi \sqrt{3f}$, where $y(t)$ is given by:

$$\dot{y}_{ifb}^{TC}(t) = \frac{\beta \psi [2\alpha - (\beta - \gamma) \sum_i \omega_{ifb}^{TC}]}{2\beta - \gamma} - \varphi \sum_i v_{ifb}^{TC}(t), \quad y(0) = 0 \quad (16)$$

From (13)-(16), we see that the transfer prices, the consumer prices and the abatement efforts under contingent strategies have a similar pattern to those under commitment strategies. The consumer prices are constant over time, yet the abatement efforts should increase along with the pollution stock. A noteworthy observation is that although the pollution stock is payoff-irrelevant to the retailers, retailers' pricing strategies are not equivalent between commitment and contingent strategies. This is due to the strategic interaction between the manufacturers in terms of abatement efforts during the second stage of the game. In fact, the retailers set the constant consumer prices regardless of the pollution accumulation and, as shown from (11) and (15), the retailers' optimal responses remain qualitatively the same regardless of the strategy type. However, manufacturers' transfer prices vary depending on whether they revise abatement strategies depending upon the pollution stock, because interaction between competing manufacturers generates different state trajectories. This in turn influences consumer prices of the retailers indirectly.

4.3 Horizontal integration and vertical competition

This subsection combines horizontal integration and vertical competition where the manufacturers and the retailers centralize their decisions separately to maximize their horizontal profits, as in a bilateral monopoly. In this case, there is horizontal internalization of pollution externalities by the manufacturers and horizontal coordination of substitution effects on demand by the retailers. However, there is no vertical coordination and double marginalization prevails while pollution externalities are not internalized in retailers' consumer prices. As in subsection 4.2, a two-stage game approach is followed.

Theorem 4. *Under commitment strategies (OLNE), manufacturer i 's transfer price and abatement effort and retailer i 's consumer price are unique and given by:*

$$\omega_{iol}^{HIVC} = \left[\frac{\delta T((\beta - \gamma)\psi^2 + 2\varphi^2) - (\beta - \gamma)\psi^2 \text{th}(\delta T)}{\delta T((\beta - \gamma)\psi^2 + 4\varphi^2) - (\beta - \gamma)\psi^2 \text{th}(\delta T)} \right] \frac{\alpha}{(\beta - \gamma)} \quad (17)$$

$$v_{iol}^{HIVC}(t) = \frac{\alpha \delta \varphi T \psi}{[\delta T((\beta - \gamma)\psi^2 + 4\varphi^2) - (\beta - \gamma)\psi^2 \text{th}(\delta T)]} \left[1 - \frac{\text{ch}(\delta t)}{\text{ch}(\delta T)} \right] \quad (18)$$

$$p_{iol}^{HIVC}(t) = \left[1 + \frac{\delta T((\beta - \gamma)\psi^2 + 2\varphi^2) - (\beta - \gamma)\psi^2 \text{th}(\delta T)}{\delta T((\beta - \gamma)\psi^2 + 4\varphi^2) - (\beta - \gamma)\psi^2 \text{th}(\delta T)} \right] \frac{\alpha}{2(\beta - \gamma)} \quad (19)$$

$i = 1, 2$, where the superscript HIVC refers to "horizontal integration and vertical competition," and $\delta \equiv 2\varphi \sqrt{f}$. Accordingly, the pollution stock is unique and given by:

$$y_{iol}^{HIVC}(t) = \frac{2\alpha \varphi^2 T \psi}{[(\beta - \gamma)\psi^2 + 4\varphi^2] \delta T - (\beta - \gamma)\psi^2 \text{th}(\delta T)} \frac{\text{sh}(\delta t)}{\text{ch}(\delta T)} \quad (20)$$

From (17) and (19), the transfer prices and the consumer prices under commitment strategies are both

constant as in total competition case. From (18), manufacturer i 's abatement efforts are also initially high and concavely decreasing until zero, and from (20), the pollution stock increases over time.

Lemma 1. *In horizontal integration and vertical competition case, commitment strategies are equivalent to contingent strategies for both manufacturers and retailers.*

The equivalence between commitment and contingent strategies results not only from the fact that the pollution stock is payoff-irrelevant to the retailers, but also from the absence of horizontal competition, and hence of strategic interaction between the manufacturers in terms of abatement efforts or between the retailers in terms of consumer prices during the second stage of the game. These imply that when firms coordinate their decisions horizontally within each tier, manufacturers and retailers may choose either strategy, committing strategies with each other or continuously revising them based on the pollution stock depending on the circumstances, e.g., resources required for each strategy and the availability of the information regarding pollution stock.

4.4 Horizontal competition and vertical integration

This subsection combines horizontal competition and vertical integration where manufacturer i and retailer i , $i = 1, 2$, centralize their vertical decisions to maximize their vertical profits, as in a duopoly. In this case, there is vertical coordination because double marginalization is totally mitigated, but no horizontal coordination due to improper internalization of pollution externalities by the manufacturers and price competition for demand by the retailers. The pollution externalities are only vertically internalized in the supply chains decisions.

Theorem 5. *Under commitment strategies (OLNE), manufacturer i 's abatement effort and retailer i 's consumer price are unique and given by:*

$$v_{iol}^{HCVI}(t) = \frac{\alpha\beta\varphi\psi}{(\beta-\gamma)^2\psi^2+\varphi^2(2\beta-\gamma)} \left[1 - \frac{\text{ch}(\vartheta t)}{\text{ch}(\vartheta T)} \right] \quad (21)$$

$$p_{iol}^{HCVI}(t) = \left(\frac{\beta-\gamma}{2\beta-\gamma} \right) \left\{ \frac{\alpha}{\beta-\gamma} + \frac{\alpha\beta\psi^2}{(\beta-\gamma)^2\psi^2+\varphi^2(2\beta-\gamma)} \left[1 - \frac{\text{ch}(\vartheta t)}{\text{ch}(\vartheta T)} \right] \right\} \quad (22)$$

$i = 1, 2$, where the superscript HCVI refers to "horizontal competition and vertical integration" and

$\vartheta \equiv \sqrt{\frac{2f[(\beta-\gamma)^2\psi^2+\varphi^2(2\beta-\gamma)]}{2\beta-\gamma}}$. Accordingly, the pollution stock is unique and given by:

$$y_{ol}^{HCVI}(t) = \frac{\sqrt{2}\alpha\beta\psi}{\sqrt{f(2\beta-\gamma)[(\beta-\gamma)^2\psi^2+\varphi^2(2\beta-\gamma)]}} \frac{\text{sh}(\vartheta t)}{\text{ch}(\vartheta T)} \quad (23)$$

In (22), the retailers' consumer prices are concavely decreasing over time, which implies convexly increasing demands. From (21), abatement efforts follow the same pattern as the retailers' consumer prices, i.e., concavely decreasing until zero. From (23), the pollution stock is strictly increasing over time. Similar to the total integration case, manufacturer's abatement efforts and retailer's consumer prices are positively associated. These results imply that patterns of pricing and abatement decisions are heavily influenced by vertical market configurations compared to horizontal market configurations.

Theorem 6. Under contingent strategies (FBNE), manufacturer i 's abatement effort and retailer i 's consumer price are:

$$v_{ifb}^{HCVI}(t) = \frac{\varphi(2\beta-\gamma)f}{\sigma} \text{th}\left(\frac{\sigma(T-t)}{2\beta-\gamma}\right) y(t) + \frac{\alpha\varphi f\psi(4\beta^2-3\beta\gamma+\gamma^2)}{\sigma^2} \left[1 - \text{sech}\left(\frac{\sigma(T-t)}{2\beta-\gamma}\right)\right] \quad (24)$$

$$p_{ifb}^{HCVI}(t) = \frac{\alpha}{2\beta-\gamma} + f\psi(\beta-\gamma) \left\{ \frac{1}{\sigma} \text{th}\left(\frac{\sigma(T-t)}{2\beta-\gamma}\right) y(t) + \frac{\alpha\psi(4\beta^2-3\beta\gamma+\gamma^2)}{\sigma^2(2\beta-\gamma)} \left[1 - \text{sech}\left(\frac{\sigma(T-t)}{2\beta-\gamma}\right)\right] \right\} \quad (25)$$

$i = 1, 2$, $\sigma \equiv \sqrt{f[6\beta^3\psi^2 + 2\beta\gamma(5\gamma\psi^2 - 6\varphi^2) - 2\beta^2(7\gamma\psi^2 - 6\varphi^2) - \gamma^2(2\gamma\psi^2 - 3\varphi^2)]}$, where $y(t)$ is given by:

$$y_{ifb}^{HCVI}(t) = \frac{2\alpha\beta\psi}{2\beta-\gamma} - \left[\frac{(\beta-\gamma)^2\psi^2}{(2\beta-\gamma)\varphi} + \varphi \right] \sum_i v_{ifb}^{HCVI}(t), \quad y(0) = 0 \quad (26)$$

In this configuration, strategic interaction involves the two vertically integrated chains that compete on prices and make non-cooperative abatement efforts. As a consequence, the resulting pollution stock follows different trajectories depending on the strategy type, leading to distinct pricing and abatement efforts between commitment and contingent strategies.

5. Comparative analysis

Based on our previous results, we now assess both analytically and numerically the relative impact of horizontal and vertical competition, on the one hand, and strategy types, on the other hand, on the pricing and abatement strategies, the pollution stock and the profits generated by the two chains.

5.1 Analytical comparisons

Theorem 7. In terms of the terminal pollution stock,

- (i) In vertically integrative supply chains, horizontal integration results in the lower terminal pollution stock than horizontal competition under commitment strategy, i.e., $y_{ol}^{HCVI}(T) > y^{TI}(T)$.
- (ii) In horizontally integrative supply chains, vertical integration results in the larger terminal pollution stock than vertical competition, i.e., $y^{TI}(T) > y^{HIVC}(T)$.

In the TI case and the HCVI case, the two supply chains are vertically integrated, and the only difference is whether they are horizontally competing or integrated. By comparing these cases, the effect of horizontal structure in the case of vertical integration can be isolated. Theorem 7(i) implies that horizontal competition leads to more accumulated pollution than horizontal integration, assuming that the two supply chains are vertically integrated. The rationale behind this is that price competition decreases sales prices, as shown in the subsequent numerical illustrations, and increases the sales volume, which raises the polluting emissions. Furthermore, pollution externalities are only partially internalized, which lessens the incentive of each manufacturer to invest in abatement efforts.

On the other hand, in the TI case and the HIVC case, the two supply chains are horizontally integrated, and the only difference is whether they are vertically competing or integrated. By comparing these cases, the influence of vertical structure in the horizontal integration case can be

isolated. Theorem 7(ii) states that vertical competition results in lower pollution stock than vertical integration, assuming that industries are horizontally integrated. A possible explanation is that the double marginalization reduces the sales and then the emissions and it also promotes manufacturers' incentives for abatement efforts, which will be confirmed in the subsequent numerical results.

Note that the underlying drivers of the environmental effects in Theorem 7 are in fact related to the two factors: output volume and incentives for abatement. We term these drivers as 'volume effect' and 'polluting effect' respectively, and investigate them in detail in the subsequent section, showing that each effect can be more pronounced under distinct configurations.

Theorem 8. *In vertically competitive supply chains, there exists $\bar{\gamma} > 0$ such that for any $\gamma > \bar{\gamma}$, horizontal integration results in the lower terminal pollution stock than horizontal competition under commitment strategy, i.e., $y_{ol}^{TC}(T) > y^{HIVC}(T)$.*

In TC and HIVC, the two supply chains are vertically competing, and the only difference is whether they are horizontally competing or integrated. By comparing these cases, the influence of horizontal structure in the case of vertical competition can be isolated. Theorem 8 implies that horizontal competition will lead to more accumulated pollution than horizontal integration, assuming that the supply chains are vertically competing and that price competition prevails. Though Theorem 7 assumes vertical integration and Theorem 8 assumes vertical competition, both theorems isolate the effect of horizontal competition and horizontal integration and therefore the underlying mechanisms are fundamentally very similar. Note that Theorem 8 states a sufficient condition, that is, $\gamma > \bar{\gamma}$, for $y_{ol}^{TC}(T) > y^{HIVC}(T)$ to hold. For $0 \leq \gamma \leq \bar{\gamma}$, however, $y_{ol}^{TC}(T)/y^{HIVC}(T) > 1$ still holds in our subsequent numerical analysis. For reference, the best estimated minimum of $y_{ol}^{TC}(T)/y^{HIVC}(T)$ from Mathematica with constraints $\beta, \psi, \phi, f, T > 0, 0 \leq \gamma \leq \bar{\gamma}$ is larger than 1 (i.e., 1).

Theorem 9. *In horizontally competitive supply chains, there exists $\bar{\phi} > 0$ such that for any $\phi > \bar{\phi}$, vertical integration results in the larger terminal pollution stock than vertical competition under commitment strategy, i.e., $y_{ol}^{HCVI}(T) > y_{ol}^{TC}(T)$.*

In the TC and the HCVI cases, the two supply chains are horizontally competing, and the only difference is whether they are vertically competing or integrated. Therefore, by comparing these cases, the effect of vertical structure can be isolated in the case of horizontal competition. Assuming that industries are horizontally competing and have sufficiently effective abatement efforts, Theorem 9 implies that vertical competition leads to less pollution than vertical integration. Though Theorem 7 assumes horizontal integration and Theorem 9 assumes horizontal competition, both theorems suggest that the underlying mechanisms are fundamentally similar. Theorem 9 states a sufficient condition for $y_{ol}^{HCVI}(T) > y_{ol}^{TC}(T)$. For $0 < \phi \leq \bar{\phi}$, however, $y_{ol}^{HCVI}(T)/y_{ol}^{TC}(T) > 1$ still holds in our numerical analysis. For reference, the minimum of $y_{ol}^{HCVI}(T)/y_{ol}^{TC}(T)$ from Mathematica with constraints $\beta, \gamma, \psi, f, T > 0$ and $0 < \phi \leq \bar{\phi}$ is larger than 1 (i.e., 2).

Theorem 10. *There exist $\tilde{\beta} > 0$ and $\tilde{\varphi} > 0$ such that for any $\beta > \tilde{\beta}$ and $\varphi > \tilde{\varphi}$, perfect integration horizontally and vertically results in the larger terminal pollution stock than no integration under commitment strategy, i.e., $y^{TI}(T) > y_{ol}^{TC}(T)$.*

In contrast to Theorem 7 and to Theorem 9, Theorem 10 compares the relative effects of horizontal structure and vertical structure. The TI configuration combines vertical integration and horizontal integration, while the TC configuration relates vertical competition and horizontal competition. For sufficiently large values of marginal sensitivity of retailer i 's demand to own price and marginal effectiveness of abatement efforts, the pollution stock is greater in the TI than the TC, implying that the relative impact of vertical integration on pollution accumulation is stronger than that of horizontal integration. Although internalization of pollution externalities between manufacturers and horizontal coordination of substitution effects between retailers drive lower pollution stock, the elimination of double marginalization may increase the pollution stock more rapidly, making the perfect coordination more polluting than no coordination.

	$\gamma < \bar{\gamma}$	$\bar{\gamma}$	$\gamma > \bar{\gamma}$
S ^ S	HIVC < TI TI, TC < HCVI		HIVC < TI, TC < HCVI
S V S	HIVC < TI < HCVI		HIVC < TI, TC TI < HCVI

Tab.3. Influence of horizontal and vertical competition on pollution accumulation under commitment strategies. Based on the previous theorems, Table 3 summarizes the combinative effect of horizontal and vertical competition on pollution accumulation as follows. Overall, when price competition and effectiveness of abatement efforts are both relatively high ($\gamma > \bar{\gamma}, \varphi > \bar{\varphi}$), horizontally competing and vertically integrated supply chains generate the highest pollution stock while horizontally integrated and vertically competing supply chains result in the lowest pollution stock. The pollution stock of the total integration and total competition cases are in between. These relationships may hold in other conditions as well. Furthermore, when the sensitivity of demand to the firm's own price is relatively high ($\beta > \tilde{\beta}$) and effectiveness of abatement efforts is sufficiently high ($\varphi > \tilde{\varphi}$), the relative effect of vertical structure (i.e., vertical competition vs. vertical integration) is more likely to outweigh the relative effect of horizontal structure (i.e., horizontal competition vs. horizontal integration).

5.2 Numerical comparisons

We conduct numerical study to further understand the underlying mechanisms regarding the impact of competition on pollution accumulation. The parameter values for the numerical study are chosen based on the apparel industry. The apparel industry has been known as a highly polluting industry (Conca, 2016). In this industry, curbing the manufacturing emissions are important because most of

polluting emissions are generated in the manufacturing stage. It has been reported that manufacturing one T-shirt generate 4.8kg CO_{2e} (Muthu, 2014; p.166), therefore we set $\psi = 5$. We specify the market potential (α) as 100 and the price sensitivity (β) as 5, in accordance with the studies for the apparel industry (e.g., Marti *et al.*, 2015; Ghosh and Shah, 2012). The cross-price sensitivity (γ) is specified as 1 to satisfy $\gamma < \beta$. We assume that $\varphi = 15$ and $f = 0.1$ to ensure that firms have nonnegative profit. Note that, though $\varphi > \psi$, since the abatement cost increases quadratically in v_i for each manufacturer while the revenue increases in demand linearly, increasing output can be considerably polluting due to abatement cost burden. The planning horizon is 12 months ($T = 12$) due to short finite horizon. We consider a broad set of parameter values for $\alpha, \beta, \gamma, \varphi, f, \psi$ to ensure the robustness of the numerical results. The base case values and the ranges of parameter values used in the numerical study are summarized in Table 4. In particular, we examine diverse situations with regard to the level of abatement difficulty as shown in the wide range of the parameter values of φ and ψ in the table. Similarly, we also consider various situations regarding pollution burden as shown in the broad range of the parameter values of f . For brevity, we summarize the major results of the numerical analysis, focusing on the robustness of the insights under the wide sets of parameters.

Parameter	α	β	γ	φ	f	ψ
Base case	100	5	1	15	0.1	5
Range	[10,1000]	[1,4,27]	[0,3,3]	[8,30]	[0.01,0.38]	[0.1,10]

Tab. 4. Base case and range values

Comparisons involving the cooperative, commitment and contingent strategies on a numerical basis are provided hereafter. We first illustrate the pricing strategies and demand (section 5.2.1) and the abatement strategies (section 5.2.2) in each market structure. We then discuss their impacts on the pollution accumulation (section 5.2.3) and the implications on the supply chain profits (section 5.2.4).

5.2.1. Pricing strategies

In this subsection, we examine the effects of competition and the strategy type on prices. We then elaborate the implications on demand, which also relates to the ‘volume effect’ on the pollution stock.

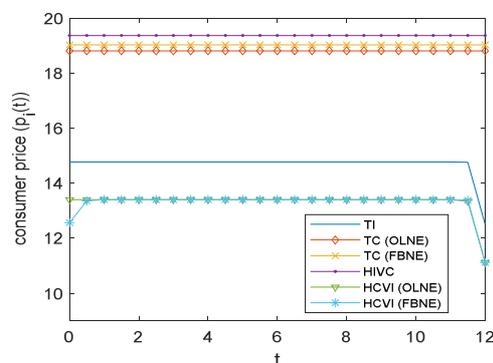


Fig. 2. Retailer i 's consumer price ($p_i(t)$)

Figure 2 illustrates the evolution of consumer prices over time in the base case. Throughout the planning horizon, the case of horizontal integration and vertical competition (HIVC) results in the

highest consumer price, followed by total competition (TC), total integration (TI) and horizontal competition and vertical integration (HCVI) regardless of the strategy types. To compare the impacts of competition on the consumer prices more precisely, we also examine the average consumer price over the planning period (i.e., $\bar{p}_i = \frac{1}{T} \int_0^T p_i(t) dt$). In line with Figure 2, the average consumer price (\bar{p}_i) has the following order, i.e., $\bar{p}_i^{HIVC} > \bar{p}_i^{TC} > \bar{p}_i^{TI} > \bar{p}_i^{HCVI}$. The optimal pricing strategies and the consequent demand in the base case are reported in Table 5, from which we make several observations as follows.

First, the impacts of strategy types on the pricing strategy may vary significantly depending on the market configurations. Whether firms must set higher or lower prices under commitment strategy compared to under contingent strategy is determined by market configurations. Although a contingent strategy results in higher consumer price and smaller demand than commitment strategy in the TC case, it leads to lower price and greater demand in the HCVI case. The reason is that, different from the HCVI case, the TC case exhibits the double marginalization effect, thus strategic interactions between manufacturers result in higher transfer prices, leading to higher consumer price. The effects of competition on consumer prices do not depend heavily on the strategy type. We observe that the same order of consumer prices holds for all parameter ranges across commitment and contingent strategies, except for $\gamma < 0.5$ or $\psi > 8.2$ where TC yields higher \bar{p}_i than HIVC due to relatively low competitive intensity and high transfer price under contingent strategy. The cumulative demand D_{cum} is thus generally the largest in the case of HCVI, followed by TI, TC and HIVC.

Second, strategy types may also change the equilibrium path of price. In HCVI configuration, retailers' prices under commitment strategies are concavely decreasing in line with Theorem 5. In contrast, prices under contingent strategies first increase and then decrease over time. That is, while sales prices are at their highest at the beginning under commitment strategies, they are at highest during the intermediate periods under contingent strategies. These impacts can be more apparent under lower f , where the lessened pollution burden causes more dynamic price adjustment of firms.

	TI	TC (OLNE)	TC (FBNE)	HIVC	HCVI (OLNE)	HCVI (FBNE)
\bar{p}_i	14.75	18.81	19.03	19.37	13.38	13.37
ω_i	n/a	13.87	14.25	13.74	n/a	n/a
$D_{cum} = \sum_i \int_0^T D_i(t) dt$	983.55	593.81	573.26	540.47	1115.9	1116.4

Tab. 5. Pricing strategies in the base case

Third, the transfer prices ω_i are lower in HIVC case than in TC case under all ranges of parameters. Given that the retailers in HIVC would charge relatively higher sales price due to the internalization of substitution effects, which leads to lower demand, the manufacturers have less room to further raise the transfer prices. This implies that the retailers in HIVC case get higher profit margins than in TC case. In other words, horizontal integration strengthens double marginalization for retailers' benefit.

The numerical results also confirm discussions on Theorems 7-9 by showing that horizontal competition generally leads to lower consumer prices than horizontal integration, when the market is

either vertically integrative or competitive, and that vertical competition increases consumer prices regardless of the horizontal structure. Under horizontal competition, retailers do not fully internalize the substitution effects between demands, leading to more competitive consumer prices. In contrast, the overpricing strategies under vertical competition are mainly due to the double marginalization. Therefore, when the market structure changes either from horizontal integration to horizontal competition, or from vertical competition to vertical integration, the sales volume increases due to lower prices, and the gross volume of emissions enlarges as a consequence, which implies a ‘*volume effect*’ in both cases, i.e., ‘horizontally-induced volume effect’ and ‘vertically-induced volume effect’. Note that the vertically-induced volume effect is generally greater than the horizontally-induced volume effect. That is, volume effect is influenced more significantly by vertical market configurations. Table 6 shows that while both horizontal competition and vertical integration generally leads to relatively higher D_{cum} compared to horizontal integration and vertical competition respectively, the impacts of vertical configuration is greater than the impacts of horizontal configuration. For instance, when α varies from 10 to 1000, horizontal competition leads to D_{cum} which is 106~114% of that of horizontal integration, while vertical integration leads to D_{cum} which is 182~195% of that of vertical competition. Both horizontally-induced and vertically-induced volume effects become greater under smaller β and greater γ , implying that the consumers’ buying behaviors influence the degree of the volume effects significantly. The effects of strategy types on the volume effects also depend on the configurations. While the horizontally-induced volume effect can be more prominent under commitment strategies under vertical competition, the vertically-induced volume effect is intensified under contingent strategies under horizontal competition.

Impact		Strategy	α	β	γ	φ	f	ψ
Horizontal competition vs. horizontal integration	under VI*: $D_{cum}^{HCVI}/D_{cum}^{TI}$	OLNE	[1.13,1.13]	[1.57,1.03]	[1.00,1.56]	[1.16,1.12]	[1.14,1.13]	[1.11,1.17]
		FBNE	[1.14,1.14]	[1.57,1.04]	[1.00,1.56]	[1.16,1.12]	[1.14,1.13]	[1.11,1.17]
	under VC*: $D_{cum}^{TC}/D_{cum}^{HIVC}$	OLNE	[1.10,1.10]	[1.55,1.01]	[1.00,1.46]	[1.09,1.11]	[1.10,1.10]	[1.11,1.08]
		FBNE	[1.06,1.06]	[1.54,0.89]	[0.96,1.43]	[1.02,1.10]	[1.06,1.06]	[1.11,0.98]
Vertical integration vs. vertical competition	under HI*: $D_{cum}^{TI}/D_{cum}^{HIVC}$		[1.82,1.82]	[1.98,1.42]	[1.78,1.91]	[1.67,1.95]	[1.82,1.82]	[2.00,1.55]
	under HC*: $D_{cum}^{HCVI}/D_{cum}^{TC}$	OLNE	[1.90, 1.90]	[2.01,1.45]	[1.79,2.04]	[1.78,1.97]	[1.89,1.88]	[2.00,1.69]
		FBNE	[1.95,1.95]	[2.03,1.66]	[1.86,2.09]	[1.91,1.98]	[1.95,1.95]	[2.00,1.87]

*) VI (vertical integration), VC (vertical integration), HI (horizontal integration), HC (horizontal competition)

Tab. 6. Impacts of competition on the cumulative demand

5.2.2. Abatement strategies

In this subsection, we investigate the effects of competition and the strategy type on abatement efforts. We also discuss the manufacturer’s incentives on abatement, which relates to the ‘polluting effects’. Figure 3 depicts the evolution of abatement efforts over time in the base case. Among the configurations, HCVI generally yields the largest abatement efforts, followed by TI, TC and HIVC. In terms of the average abatement efforts over the planning horizon (i.e., $\bar{v}_i = \frac{1}{T} \int_0^T v_i(t) dt$), it also holds that $\bar{v}_i^{HCVI} > \bar{v}_i^{TI} > \bar{v}_i^{TC} > \bar{v}_i^{HIVC}$ (Table 7). The low level of manufacturers’ abatement efforts in

HIVC is adequately reflected in the corresponding low level of manufacturers' transfer prices. The same rank also holds for most of parameter ranges in both commitment and contingent strategies.

Different from the effects of strategy types on prices, the commitment strategy yields higher abatement efforts and abatement rates compared to the contingent strategy in either TC or HCVI case, confirming that the manufacturing process becomes cleaner under the commitment strategy whenever the manufacturers are competitive. An intuitive explanation can be that contingent strategy where abatement efforts of competing manufacturers are continuously revised intensify the effects of imperfect internalization of pollution externalities, resulting in lower pollution abatement. Strategy types may also influence the equilibrium path of abatement efforts under horizontal competition. Similar to pricing paths, abatement efforts follow concavely decreasing trajectories under commitment strategies in line with Theorems 2 and 5, being at their highest levels during the initial periods under commitment strategies, whereas they become the highest during the intermediate periods under contingent strategies. These impacts can be more prominent under lower f , where firms have relatively low incentives to reduce emissions.

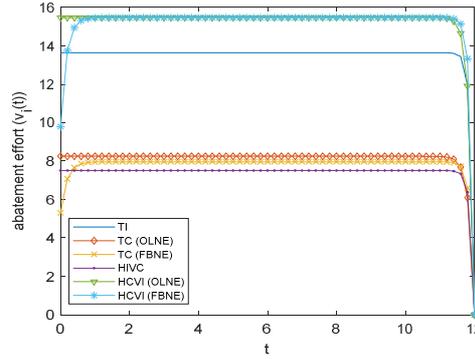


Fig. 3. Manufacturer i 's abatement efforts ($v_i(t)$)

The numerical results show that the market configuration with lower sales prices generally yields larger abatement. For instance, HCVI which exhibits the lowest sales price yields the largest abatement efforts. This relationship is intuitive in that lower sales prices enlarge the sales demand and thus gross emissions volume, which necessitates more abatement efforts. However, whether the largest abatement efforts in HCVI are driven by the manufacturer's incentive for abatement for a given demand level, or whether they are merely driven by the largest volume is not straightforward.

We thus introduce the abatement rate (ξ), which measures the average abatement efforts per demand, i.e., $\xi_i = \int_0^T v_i(t) dt / \int_0^T D_i(t) dt$, to examine the impacts of competition on the manufacturer's incentives for abatement efforts more precisely. Table 7 shows that $\xi_i^{HIVC} > \xi_i^{TI} > \xi_i^{TC} > \xi_i^{HCVI}$ holds under either strategy type in the base case. In all ranges of parameter values, it also holds that ξ_i^{HIVC} is the largest and ξ_i^{HCVI} is the smallest. This implies that when the demand level is taken into account, the manufacturers have the highest abatement incentive in HIVC and the lowest incentive in HCVI.

More specifically, horizontal competition leads to lower abatement rates for a given vertical structure. Since the pollution externalities are not internalized by competitive manufacturers, the manufacturers'

abatement incentives decrease accordingly. On the other hand, vertical integration leads to lower abatement rates than vertical competition for a given horizontal configuration. A possible explanation is that vertically integrated chains can control pricing and abatement decisions simultaneously, thus having opportunities to partially replace costly abatement efforts with adjustment of demand.

	TI	TC (OLNE)	TC (FBNE)	HIVC	HCVI (OLNE)	HCVI (FBNE)
\bar{v}_i	13.53	8.14	7.85	7.44	15.29	15.25
$\xi_i = \int_0^T v_i(t) dt / \int_0^T D_i(t) dt$	0.3301	0.3292	0.3284	0.3304	0.3288	0.3279

Tab. 7. Abatement strategies in the base case

As horizontal competition and vertical integration lead to relatively lower abatement rates respectively, the degree of net emissions for a given demand increases, implying ‘horizontally-induced polluting effect’ and ‘vertically-induced polluting effect’. Table 8 shows that contrary to volume effects, the horizontally-induced polluting effect is generally greater than the vertically-induced polluting effect. That is, the polluting effect depends more on horizontal market configurations. For instance, while horizontal competition reduces ξ_i by 0.4~0.7% compared to horizontal integration over the range of parameter α , vertical integration reduces ξ_i by 0.1~0.2% compared to vertical competition. The polluting effects become more evident under lower φ , lower f and higher ψ , implying that difficulties in abatement and the government pressures influence the degree of the polluting effects. Different from volume effects under which the effects of strategy types depend on market configuration, polluting effects, either horizontally-induced or vertically-induced, are generally intensified under contingent strategies, which result in relatively lower abatement rates.

Impact		Strategy	α	β	γ	φ	f	ψ
Horizontal competition vs. horizontal integration	under VI [*] : $\xi_i^{HCVI} / \xi_i^{TI}$	OLNE	[0.996,0.996]	[0.996,0.995]	[0.996,0.996]	[0.994,0.998]	[0.988,0.998]	[0.996,0.996]
		FBNE	[0.993,0.993]	[0.994,0.991]	[0.993,0.994]	[0.989,0.997]	[0.978,0.997]	[0.994,0.992]
	under VC [*] : $\xi_i^{TC} / \xi_i^{HIVC}$	OLNE	[0.996,0.996]	[0.996,0.996]	[0.996,0.996]	[0.995,0.998]	[0.988,0.998]	[0.996,0.996]
		FBNE	[0.994,0.994]	[0.994,0.994]	[0.994,0.994]	[0.991,0.997]	[0.981,0.997]	[0.994,0.994]
Vertical integration vs. vertical competition	under HI [*] : $\xi_i^{TI} / \xi_i^{HIVC}$		[0.999,0.999]	[0.999,0.995]	[0.999,1.000]	[0.997,1.000]	[0.997,1.000]	[1.000,0.997]
	under HC [*] : $\xi_i^{HCVI} / \xi_i^{TC}$	OLNE	[0.999,0.999]	[0.999,0.993]	[0.998,1.000]	[0.996,1.000]	[0.997,0.999]	[1.000,0.996]
		FBNE	[0.998,0.998]	[0.999,0.992]	[0.998,0.999]	[0.995,1.000]	[0.995,0.999]	[1.000,0.995]

*) VI (vertical integration), VC (vertical integration), HI (horizontal integration), HC (horizontal competition)

Tab. 8. Impacts of competition on the abatement rates

5.2.3. The environmental performance

Then, what are the comprehensive impacts of competition on the final pollution stock considering the pricing and abatement strategies? Table 9 shows that $y^{HIVC}(T) < y^{TC}(T) < y^{TI}(T) < y^{HCVI}(T)$ holds in the base case. The numerical analysis also confirms that the final pollution stock is the largest in HCVI, the smallest in HIVC, and intermediate in TI and TC for all ranges of parameter values regardless of strategy types, which also supports Theorems 7-10. While the TI case generally results in the larger $y(T)$ than the TC, the opposite may also hold under small β or large γ where the competitive intensity is relatively large and thus the impact of horizontal internalization becomes influential. These results imply that horizontal competition and vertical integration result in the larger terminal pollution stock respectively as a result of the volume effects and the polluting effects.

One important question remaining here is whether the environmentally detrimental effect of horizontal competition or vertical integration remain even after its higher demand is taken into account. We thus examine the pollution intensity (PI), which is measured as the terminal pollution stock per unit of cumulative demand, i.e., $PI = y(T)/\sum_i \int_0^T D_i(t)$, between the four configurations. Table 9 shows that $PI^{HIVC} < PI^{TI} < PI^{TC} < PI^{HCVI}$ holds for a given strategy type in the base case. The numerical analysis for all ranges of parameter values also confirms that for a given strategy type, the pollution intensity is the highest in HCVI and the lowest in HIVC. In terms of the intermediate ranks, TC leads to larger pollution intensity than TI in most cases, except for $\beta \geq 19.5$. These results thus confirm the environmental demerits of horizontal competition and vertical integration.

	TI	TC (OLNE)	TC (FBNE)	HIVC	HCVI (OLNE)	HCVI (FBNE)
$y(T)$	47.67	36.88	42.10	23.74	75.68	91.43
PI	0.049	0.062	0.073	0.044	0.068	0.082
AI	6.81	5.30	4.47	7.52	4.85	4.00

Tab. 9. The environmental performance in the base case

When horizontal competition prevails, the contingent strategies result in the larger terminal pollution stock and the larger pollution intensity compared to the commitment strategies, because manufacturers behave more opportunistically when they are able to update their abatement decisions. Compared to TC, the polluting effect of contingent strategies is stronger under HCVI because of lower abatement, and also lower price, hence more gross emissions. Thus, contingent strategies differ from commitment strategies in that they lead both to higher emissions and greater savings on abatement efforts, but also to higher pollution costs.

Table 9 shows that the abatement intensity, denoted by AI and measured as cumulative abatement efforts per unit of terminal pollution stock, i.e., $AI = \sum_i \int_0^T v_i(t)/y(T)$, has the following order, i.e., $AI^{HIVC} > AI^{TI} > AI^{TC} > AI^{HCVI}$, for a given strategy type. As expected, it is the same as the one for abatement rates and the opposite to the one for pollution intensity, supporting the horizontally-induced and vertically-induced polluting effects. The sensitivity analysis shows that the abatement intensity is the largest in HIVC and the smallest in HCVI under FBNE strategies for all parameter ranges. In between, TI generally has higher abatement intensity than TC except for $\beta \geq 19.5$.

5.2.4. Supply chain profits

In this subsection, we discuss the effects of competition and strategy types on supply chain profits. Let J, J^m, J^r denote the supply chains', the manufacturers', and the retailers' total profits respectively. In the base case, $J^{TI} > J^{HCVI} > J^{TC} > J^{HIVC}$ holds for both strategy types (Table 10).

	TI	TC (OLNE)	TC (FBNE)	HIVC	HCVI (OLNE)	HCVI (FBNE)
J	12,294	10,361	9,904.3	9,798.8	12,064	10,695
J^m	8,389.3	7,422.6	7,165.8	6,755.9	8,274.0	7,561.1
J^r	3,905.1	2,938.4	2,738.6	3,042.8	3,789.8	3,133.9

Tab. 10. Supply chain profits in the base case

It is straightforward that the TI case, where all firms centralize their decisions, yields the largest supply chains' profits. Apart from TI, the sensitivity analysis shows that the orders between HCVI, TC and HIVC depends on the parameter values. For instance, $J^{TI} > J^{HCVI} > J^{TC} > J^{HIVC}$ holds for $0 \leq \gamma < 2.2$, $J^{TI} > J^{TC} > J^{HCVI} > J^{HIVC}$ for $2.2 \leq \gamma < 3$, and $J^{TI} > J^{TC} > J^{HIVC} > J^{HCVI}$ for $\gamma \geq 3$. It is noteworthy that the impacts of vertical configurations on supply chains' profits significantly depends on the horizontal configurations, and vice versa. To be specific, horizontal competition results in lower supply chains' profits than horizontal integration due to intense price competition when the firms are vertically integrative. Similarly, when the firms are horizontally integrative, vertical competition results in lower supply chains' profits than vertical integration due to the double marginalization. However, such economic impacts of horizontal (vertical) competition do not necessarily hold under vertical (horizontal) competition. For instance, the TC case mostly leads to larger supply chains' profits than the HIVC case because the demand-enhancing effect of lower price under horizontal competition becomes significant when double marginalization exists. The TC case may also yield higher profits than the HCVI case e.g., under high γ , implying that lower emissions under vertical competition can enhance the economic performance when price competition is intense. Another observation is that for a given configuration, the contingent strategy results in lower supply chains' profits than the commitment strategy for all ranges of parameter values. That is, commitment strategy has not only environmental virtue as discussed in the preceding section, but also economic merit to the entire supply chains, especially under HCVI. Note also that contingent strategies are less beneficial to both manufacturers and retailers than commitment strategies in the HCVI and TC case. This suggests that whenever contingent strategies are possible options, their related tradeoff, which is characterized by more emissions with less abatement and greater pollution costs than in commitment strategy, is detrimental to all firms.

The manufacturers' profits (J^m) and the retailers' profits (J^r) are the highest under the TI case where the Nash bargaining scheme (NBS) (see Dockner *et al.*, 2000) is used to compute J^m and J^r based on those obtained from the TC case under commitment strategy. While the manufacturers' profits are at their lowest in the HIVC case, the retailers' profits may be at their lowest in the TC case (computed with an NBS based on the TC case). That is, the HIVC case, which is the least polluting, is generally less beneficial to manufacturers than to retailers.

6. Conclusions

This research has shown that horizontal and vertical market structure has a critical influence on pollution accumulation by analyzing four market configurations using differential games models. Our results provide several insights with regard to the environmental impacts of market structures. On the one hand, vertical competition is less pollution intensive and more abatement intensive than vertical integration is. On the other hand, horizontal competition is more pollution intensive and less abatement intensive than horizontal integration does. The major mechanisms underlying these impacts

of competition are the volume effects and the polluting effects, both are vertically as well as horizontally-induced. The four configurations in Table 11 can be summarized as follows:

- Both vertical and horizontal internalization of the pollution externalities (TI) solely involve vertically-induced ‘volume effect’ and ‘polluting effect’;
- Neither vertical nor horizontal internalization of the pollution externalities (TC) exclusively involve horizontally-induced ‘volume effect’ and ‘polluting effect’;
- Horizontal internalization of the pollution externalities (HIVC) involves neither ‘volume effect’ nor ‘polluting effect;’
- Vertical internalization of the pollution externalities (HCVI) involves both vertically and horizontally-induced ‘volume effect’ and ‘polluting effect’.

Bilateral duopoly	Vertical integration	Vertical competition
Horizontal integration	<ul style="list-style-type: none"> - Intermediate consumer prices, demands, abatement efforts, and abatement rates - Intermediate terminal pollution stock - Intermediate pollution and abatement intensive - Highest supply chain profit 	<ul style="list-style-type: none"> - Highest consumer prices, lowest demands and abatement efforts, highest abatement rates - Lowest terminal pollution stock - Least pollution intensive, most abatement intensive - Lowest (intermediate) supply chain profit
Horizontal competition	<ul style="list-style-type: none"> - Lowest consumer prices, highest demands and abatement efforts, lowest abatement rates - Greatest terminal pollution stock - Most pollution intensive, least abatement intensive - Intermediate (lowest) supply chain profit - Greater terminal pollution stock and lower profits in contingent than in commitment strategy 	<ul style="list-style-type: none"> - Intermediate consumer prices, demands, abatement efforts, and abatement rates - Intermediate terminal pollution stock - Intermediate pollution and abatement intensive - Intermediate supply chain profit - Greater terminal pollution stock and lower profits in contingent than in commitment strategy

Tab. 11. Main differences between configurations

The vertically-induced volume effect is relatively larger than the horizontally-induced volume effect. That is, an increase in gross emissions in the market is derived more by the elimination of double marginalization than the introduction of competition in the market. On the contrary, the vertically-induced polluting effect is generally smaller than the horizontally-induced polluting effect, implying that the competitive pressures in the market weaken the manufacturers’ abatement incentives more significantly than vertical internalization of pollution externalities.

Strategy types also play a significant role in the context of horizontal competition. Contingent strategies result in less environment-friendly configurations under horizontal competition. The environmental demerit of contingent strategies arises mainly due to the reduced abatement incentives. The impact of contingent strategies on the gross emissions volume is modified by vertical configuration: while contingent strategies lead to smaller emissions volume under vertical competition, they result in larger emissions volume under vertical integration.

Our analysis provides valuable guidance for managers as follows.

First, we offer guidance on the profit-maximizing pricing and abatement policies which depend on the market structures and strategy types. For instance, managers need to invest relatively heavily in abatement efforts for a given output and to set a relatively higher price, thus reducing the output level, if they operate under horizontally integrated or vertically competitive markets. At the same time,

managers need to be aware that abatement and pricing policies must be revised accordingly depending on strategy types. If commitment strategies are adopted, it is advisable to increase abatement intensity, but whether to raise sales price compared to under contingent strategies depends further on market configurations. Additionally, while managers need to invest in abatement efforts most heavily during intermediate periods under contingent strategies, managers must invest actively from the beginning under commitment strategies.

Second, the growing adoption of commitment strategies in practice (e.g., Shell, BP, Gap, Nike, etc.) can be not only environmentally but also economically advantageous in the horizontally competing industries. In this context, commitment strategies serve as a measure to alleviate negative effects due to the imperfect internalization of intertemporal pollution externalities. However, in horizontally integrative industries, firms are indifferent between commitment and contingent strategies because horizontal internalization of pollution externalities and substitution effects suffice to eliminate incentives of firms to deviate from the optimal paths under commitment strategies.

Third, managers should not overlook the environmental dimension when formulating strategies that influence structural configurations in the industry. For instance, while vertical integration with supply chain partners can eliminate double marginalization, it may result in lower profits due to the increased pollution burden. Our analysis also suggests that horizontal integration, e.g., Chevron's takeover of Anadarko, in industries can lead to not only lower gross emissions but also stronger abatement incentives of firms, contradicting the argument that horizontal integration may be "just a temporary solution" for curbing emissions and results in lower greening investments (Luo et al., 2016).

Our study also offers policy advice for regulators to curb pollution emissions in industries.

First, regulators may differentiate their focus in employing policies to curb firms' emissions depending on market structures. Specifically, as polluting effects are more induced by horizontal configurations, policies to incentivize firms' abatement efforts can be preferentially implemented to horizontally competitive markets. On the other hand, as volume effects are more driven by vertical configurations, regulators may monitor whether vertically integrated chains offer excessively low prices, which cause large gross emissions as a byproduct.

Second, regulators may promote commitment strategies in horizontally competitive markets, which lead to lower pollution emissions. One indirect way is to limit the information on pollution. Although regulators typically pursue higher visibility regarding this matter, e.g., Canada's Greenhouse Gas Reporting Program publicizing information on the pollution of each facility, our results suggest that regulators may strategically restrict information visibility on the pollution based upon which firms can employ contingent strategies and revise their actions continuously.

Third, the least polluting configuration, horizontal integration and vertical competition, is more unfavorable to manufacturers than retailers. Thus, regulators can focus more on providing sufficient incentives for upstream firms to coordinate their decisions, in order to reduce environmental damage in industries.

A possible avenue for future research consists in building an empirical test to assess the relative importance of vertically and horizontally-induced volume and polluting effects in the industry.

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Appendix

Table A1. Profits under different cases and strategy types

Plugging the derived abatement and pricing strategies gives profits under commitment strategies and TI case. The payoff functions are obtained from the value functions directly under contingent strategies. The detail proofs are omitted for brevity.

Case	Strategy	Profits
TI		$J = \frac{\alpha^2}{((\beta-\gamma)\psi^2+2\varphi^2)} \left[\frac{\varphi^2 T}{\beta-\gamma} + \frac{\psi^2 \text{th}(\theta T)}{2\theta} \right]$
TC	Commitment	$J_{ol}^{m_i} = \frac{\alpha^2 \beta T^2}{2\{\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2)-\text{th}(\rho T)]+4\varphi^2\rho(2\beta-\gamma)T\}^2}$ $\left\{ 4\varphi^3\sqrt{2f} \{\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2)-\text{th}(\rho T)]+2\varphi^2\rho(2\beta-\gamma)T\} + \beta\varphi^2\psi^2 \text{sech}(\rho T)^2 [\rho \text{sh}(2\rho T) - 2\rho^2 T \text{ch}(2\rho T)] \right\}$ $J_{ol}^{r_i} = \frac{(\beta-\gamma)}{4\alpha^2\beta\varphi^4\rho^2T^3}$
	Contingent	$J_{fb}^{m_i} = \frac{\beta[\alpha-(\beta-\gamma)\omega_{ifb}^{TC}]}{3\varphi^2\tau(2\beta-\gamma)^2} \left\{ [(2\beta^2\psi^2 - 3\gamma\varphi^2 - 2\beta\gamma\psi^2 + 6\beta\varphi^2)\omega_{ifb}^{TC} - 2\alpha\beta\psi^2]\tau T + 2\beta[\alpha - (\beta - \gamma)\omega_{ifb}^{TC}] \text{th}(\tau T)\psi^2 \right\}$ $J_{fb}^{r_i} = \frac{\beta[\alpha-(\beta-\gamma)\omega_{ifb}^{TC}]^2 T}{(2\beta-\gamma)^2}$
HIVC	Commitment & Contingent	$J_{ol}^m = \frac{\delta T^2 \alpha^2 \varphi^2}{(\beta-\gamma)(\delta T(4\varphi^2+(\beta-\gamma)\psi^2)-(\beta-\gamma)\psi^2 \text{th}(\delta T))}$ $J_{ol}^r = \frac{2\alpha^2\delta^2\varphi^4T^3}{(\beta-\gamma)[((\beta-\gamma)\psi^2+4\varphi^2)\delta T-(\beta-\gamma)\psi^2 \text{th}(\delta T)]^2}$
	Commitment	$J_{ol}^i = \frac{1}{8\vartheta} \left\{ \varphi^2 \kappa^2 \text{sech}(\vartheta T)^2 [3\text{sh}(2\vartheta T) - 2\vartheta(2 + \text{ch}(2\vartheta T))T] + \frac{\kappa^2 \vartheta^2 \text{sech}(\vartheta T)^2 (2\vartheta T - \text{sh}(2\vartheta T))}{f} \right.$ $\left. + \frac{8e^{2\vartheta T} [2\vartheta T(\alpha^2\beta - 2(\beta-\gamma)^3\kappa^2\psi^2 - \alpha\gamma(\beta-\gamma)\kappa\psi) + 2\vartheta T(\alpha\beta + (\beta-\gamma)^2\kappa\psi)(\alpha - (\beta-\gamma)\kappa\psi)\text{ch}(2\vartheta T) + (\beta-\gamma)\psi(3(\beta-\gamma)^2\kappa\psi + 2\alpha\gamma)\kappa\text{sh}(2\vartheta T)]}{(1+e^{2\vartheta T})^2(2\beta-\gamma)^2} \right\}$ where $\kappa = -\frac{\alpha\beta\psi}{(\beta-\gamma)^2\psi^2+\varphi^2(2\beta-\gamma)}$
	Contingent	$J_{fb}^i = \frac{\alpha^2 f}{2\sigma^3(2\beta-\gamma)} \left\{ \sigma T(2\beta-\gamma)[6\beta\varphi^2 - (\beta+\gamma)^2\psi^2] + (4\beta^2 - 3\beta\gamma + \gamma^2)^2\psi^2 \text{th}\left(\frac{\sigma T}{2\beta-\gamma}\right) \right\}$

Proof of Theorem 1.

Skipping the time index for convenience, the corresponding joint objective criterion is:

$$\text{Max}_{p_1, p_2, v_1, v_2 \geq 0} J = \sum_i (J^{m_i} + J^i) = \int_0^T \left[\sum_{i \neq j} p_i (\alpha - \beta p_i + \gamma p_j) - \frac{\sum_i v_i^2}{2} - f y^2 \right] dt$$

subject to (5). The Hamiltonian in the total integration case is written as:

$$H = \sum_{i \neq j} p_i (\alpha - \beta p_i + \gamma p_j) - \frac{\sum_i v_i^2}{2} - f y^2 + \eta \left[\psi \sum_{i \neq j} (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i \right]$$

from which necessary conditions for optimality with symmetric controls are,

$$\dot{\eta} = 2fy, H_{v_i} = -v_i - \varphi\eta = 0 \Rightarrow v_i = -\varphi\eta, H_{p_i} = \alpha - 2\beta p_i + 2\gamma p_j - (\beta-\gamma)\eta\psi = 0 \Rightarrow p_i = \frac{\alpha - (\beta-\gamma)\eta\psi}{2(\beta-\gamma)}, i = 1, 2, i \neq j.$$

The Hamiltonian is concave in (v_1, v_2, p_1, p_2) because the Hessian is definitely negative and therefore the Legendre-Clebsch condition is satisfied (Grass *et al.*, 2008). However, it can be shown that the necessary conditions are also sufficient for optimality, which implies that the optimal solution is unique. The canonical system then writes $\dot{\eta} = 2fy, \dot{y} = \alpha\psi + ((\beta-\gamma)\psi^2 + 2\varphi^2)\eta$, which gives a general solution $\eta(t) = A_{11}e^{m_{11}t} + A_{12}e^{m_{12}t} + K_{11}$, where $m_{11} = \sqrt{2f((\beta-\gamma)\psi^2 + 2\varphi^2)}$, $m_{12} = -m_{11}$, $K_{11} = -\frac{\alpha\psi}{((\beta-\gamma)\psi^2 + 2\varphi^2)}$. Also, from $y(0) = 0$ and $\eta(T) = 0$, we have $A_{11} = \frac{-m_{11}K_{11} + 2fy_0e^{-m_{11}T}}{m_{11}(e^{m_{11}T} + e^{-m_{11}T})}$ and $A_{12} = -\frac{m_{11}K_{11} + 2fy_0e^{m_{11}T}}{m_{11}(e^{m_{11}T} + e^{-m_{11}T})}$. Plugging the derived $\eta(t)$ completes the proof. Furthermore, from (7), it can be shown that $p_i(t) \leq \frac{\alpha}{\beta-\gamma}$, $i = 1, 2$, which guarantees that the control constraints $p_i(t) \leq (\alpha + \gamma p_j(t))/\beta$ are never binding and therefore that the retailers' demands are non-negative. Strictly and convexly increasing pollution stock ensures that the control constraint of $v_i^{TI}(t)$ is never binding and therefore that the manufacturers' net current emissions (i.e., current emissions minus abatement efforts) are non-negative. □

Proof of Theorem 2. In the total competition case, the respective Hamiltonians write:

$$H^{m_i} = \omega_i (\alpha - \beta p_i + \gamma p_j) - \frac{v_i}{2} - \frac{f y^2}{2} + \lambda_i \left[\psi \sum_{i \neq j} (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i \right]$$

$$H^i = (p_i - \omega_i)(\alpha - \beta p_i + \gamma p_j) + \mu_i \left[\psi \sum_{i \neq j} (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i \right]$$

$i = 1, 2, i \neq j$, from which the equations of the costate variables are given by $\dot{\lambda}_i = -H_y^{m_i} = fy$, $\dot{\mu}_i = -H_y^{r_i} = 0$. Given $\dot{\mu}_i = 0$ and the transversality condition $\mu_i(T) = 0$, we get $\mu_i(t) = 0, i = 1, 2$. Manufacturer i 's and retailer i 's necessary conditions for optimality yield respectively $H_{v_i}^{m_i} = 0 \rightarrow v_i = -\varphi \lambda_i, H_{p_i}^{r_i} = 0 \rightarrow p_i = \frac{\alpha + \gamma p_j + \beta \omega_i}{2\beta}$. It can be shown that the necessary conditions are also sufficient for optimality, which implies that the equilibrium is unique. Since $\lambda_i = \lambda_j = \lambda$, the manufacturers' controls are symmetric, that is $v_i = v_j = -\varphi \lambda$. Also, $\omega_i = \omega_j = \omega$, for such symmetric manufacturers and therefore we get $p_i = \frac{\alpha + \beta \omega}{2\beta - \gamma}$, which is constant over time. Therefore, the canonical system writes $\dot{\lambda} = fy, \dot{y} = 2 \left[\psi \left(\alpha - \frac{(\beta - \gamma)(\alpha + \beta \omega)}{2\beta - \gamma} \right) + \varphi^2 \lambda \right]$, which has a general solution $\lambda(t) = A_{31} e^{m_{31}t} + A_{32} e^{m_{32}t} + K_{31}$ with $m_{31} = \varphi \sqrt{2f}, m_{32} = -m_{31}$ and $K_{31} = -\frac{\beta \psi [\alpha - (\beta - \gamma) \omega]}{\varphi^2 (2\beta - \gamma)}$. $A_{31} = A_{32} = \frac{-K_{31}}{(e^{m_{31}T} + e^{-m_{31}T})}$ are derived from $y(0) = 0$ and $\lambda(T) = 0$. Plugging the expressions and simplifying gives

$$J^{m_i} = \frac{\beta [\alpha - (\beta - \gamma) \omega]}{2\beta - \gamma} \left\{ \omega T + \frac{\beta [\alpha - (\beta - \gamma) \omega] \psi^2}{4m_{31} (2\beta - \gamma) \varphi^2} [-2m_{31}T + m_{31}T \operatorname{sech}(m_{31}T)^2 + \operatorname{th}(m_{31}T)] \right\}$$

$i = 1, 2$. We derive the right-hand side of J^{m_i} with respect to ω and solve to obtain, using $\rho \equiv \varphi \sqrt{2f}$:

$$\omega = \frac{\{\beta(\beta - \gamma)\psi^2[(2 - \operatorname{sech}(\rho T)^2)\rho T - \operatorname{th}(\rho T)] + 2\varphi^2\rho(2\beta - \gamma)T\} \alpha}{\{\beta(\beta - \gamma)\psi^2[(2 - \operatorname{sech}(\rho T)^2)\rho T - \operatorname{th}(\rho T)] + 4\varphi^2\rho(2\beta - \gamma)T\} (\beta - \gamma)}$$

Plugging the expression of ω and simplifying completes the proof. Furthermore, from (9), it is obvious that manufacturer i 's OLNE transfer price, ω_{iol}^{TC} , is strictly positive and lower than $\frac{\alpha}{\beta - \gamma}, i = 1, 2$, which implies, from (11), that the retailers' demands are non-negative. Strictly increasing pollution stock ensures that the manufacturers' net current emissions are non-negative. \square

Proof of Theorem 3. Since the game is symmetric, we assume that the optimal value functions are the same for the manufacturers and for the retailers, that is, $V^{m_i} = V^{m_j} = V^m$ and $V^{r_i} = V^{r_j} = V^r$. In this regard, $v_i = v_j$, we assume symmetric transfer prices such that $\omega_i = \omega_j = \omega$. We first construct the Hamilton-Jacobi-Bellman (HJB) equations:

$$-\dot{V}^m = \operatorname{Max}_{v_i} \left\{ \omega(\alpha - \beta p_i + \gamma p_j) - \frac{v_i^2}{2} - \frac{fy^2}{2} + V_y^m [\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i] \right\}$$

$$-\dot{V}^r = \operatorname{Max}_{p_i} \left\{ (p_i - \omega)(\alpha - \beta p_i + \gamma p_j) + V_y^r [\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i] \right\}$$

$i = 1, 2, i \neq j$. The maximization on the right-hand side of the HJB equations with respect to v_i and p_i gives $v_i = -\varphi V_y^m$ and $p_i = \frac{\alpha + \gamma p_j + \beta \omega - (\beta - \gamma) \psi V_y^r}{2\beta}$. Since $V^{m_i} = V^{m_j} = V^m, v_i = v_j$, and then $p_i = p_j = \frac{\alpha + \beta \omega - \psi(\beta - \gamma) V_y^r}{2\beta - \gamma}$. We make the conjectures for the value functions: $V^m(y, t) = A(t)y^2 + B(t)y + C(t), V^r(y, t) = D(t)y + E(t)$. To ensure that the HJB equations and the boundary conditions hold for all $y \geq 0$ and all t , after substituting the conjectured value functions into the HJB equations, the following set of equations should be met:

$$\dot{A} = \frac{f}{2} - 6\varphi^2 A^2, \dot{B} = -\frac{2A(3B(2\beta - \gamma)\varphi^2 + 2\psi(\beta(\alpha - \omega(\beta - \gamma)) + D(\beta - \gamma)^2\psi))}{2\beta - \gamma}$$

$$\dot{C} = -\frac{2\omega\alpha\beta - 2\omega^2\beta(\beta - \gamma) - 2\omega(\beta - \gamma)(2B\beta - D(\beta - \gamma))\psi + B(3B(2\beta - \gamma)\varphi^2 + 4\psi(\alpha\beta + D(\beta - \gamma)^2\psi))}{2(2\beta - \gamma)}$$

$$\dot{D} = -4\varphi^2 AD, \dot{E} = -\left(2BD\varphi^2 + \frac{(\beta(\alpha - \omega(\beta - \gamma)) + D(\beta - \gamma)^2\psi)(\alpha - \omega(\beta - \gamma) + D(3\beta - \gamma)\psi)}{(2\beta - \gamma)^2} \right)$$

with $A(T) = B(T) = C(T) = 0$ and $D(T) = E(T) = 0$. Solving the differential equations and plugging the solutions yields (14)-(16). Given that $y_0 = 0$, the payoff functions rewrite:

$$J^{m_i} = V^m(y_0, 0) = \frac{\beta [\alpha - (\beta - \gamma) \omega] \{ 2\beta \sqrt{3} [\alpha - (\beta - \gamma) \omega] \operatorname{th}(\varphi T \sqrt{3f}) \psi^2 - 3\varphi T \sqrt{f} [2\alpha\beta\psi^2 - (2\beta^2\psi^2 - 3\gamma\varphi^2 - 2\beta\gamma\psi^2 + 6\beta\varphi^2)\omega] \}}{9\varphi^3(2\beta - \gamma)^2\sqrt{f}}, \text{ and}$$

$$J^{r_i} = V^r(y_0, 0) = \frac{\beta [\alpha - (\beta - \gamma) \omega]^2 T}{(2\beta - \gamma)^2}. \text{ Deriving } J^{m_i} \text{ with respect to } \omega \text{ gives (13), which completes the proof.}$$

Furthermore, from (13), manufacturer i 's transfer price, ω_{ifb}^{TC} , appears to be strictly positive and lower than $\frac{\alpha}{\beta - \gamma}, i = 1, 2$, which implies, from (15), retailers' non-negative demands. Using (14) in (16), the pollution stock is strictly increasing over time because the condition $y_{fb}^{TC}(t) < \frac{\beta \psi [\alpha - (\beta - \gamma) \omega_{ifb}^{TC}] \{ 2 + \operatorname{ch}[\tau(T - t)] \}}{\tau(2\beta - \gamma) \operatorname{sh}[\tau(T - t)]}$ holds $\forall t \in [0, T]$. This ensures that manufacturers' net current emissions are non-negative. \square

Proof of Theorem 4.

In the horizontal integration and vertical competition case, the corresponding objective criteria are:

$$\text{Max}_{\omega, \omega_2, v_1, v_2 \geq 0} J^m = \int_0^T \left[\sum_{i \neq j} \omega_i (\alpha - \beta p_i + \gamma p_j) - \frac{\sum_i v_i^2}{2} - f y^2 \right] dt$$

$$\text{Max}_{p_1, p_2 \geq 0} J^r = \int_0^T \sum_{i \neq j} (p_i - \omega_i) (\alpha - \beta p_i + \gamma p_j) dt$$

subject to (5). The respective Hamiltonians write:

$$H^m = \sum_{i \neq j} \omega_i (\alpha - \beta p_i + \gamma p_j) - \frac{\sum_i v_i^2}{2} - f y^2 + \lambda [\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i]$$

$$H^r = \sum_{i \neq j} (p_i - \omega_i) (\alpha - \beta p_i + \gamma p_j) + \mu [\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i]$$

$i = 1, 2, i \neq j$, from which the equations of the costate variables are given by $\dot{\lambda} = -H_y^m = 2fy$, $\dot{\mu} = -H_y^r = 0$, which yields $\mu(t) = 0$ given the transversality condition $\mu(T) = 0$. Manufacturer's and retailer's necessary conditions for optimality are respectively $H_{v_i}^m = 0 \rightarrow v_i = -\varphi \lambda$, $H_{p_i}^r = 0 \rightarrow p_i = \frac{\alpha + 2\gamma p_j + \beta \omega_i - \gamma \omega_j}{2\beta}$. The Hamiltonians H^m and H^r are concave in (v_i, v_j) and (p_i, p_j) respectively, because the Hessians are definite negative and therefore the Legendre-Clebsch is satisfied (Grass *et al.*, 2008). It can be shown that the necessary conditions are also sufficient for optimality, which implies that the equilibrium is unique. Manufacturers' controls are symmetric, that is, $v_i = v_j = v$. Also, $\omega_i = \omega_j = \omega$, for such symmetric manufacturers and therefore we get $p_i = \frac{\alpha + (\beta - \gamma)\omega}{2(\beta - \gamma)}$, which is constant over time. The canonical system writes $\dot{\lambda} = 2fy$, $\dot{y} = \psi(\alpha - (\beta - \gamma)\omega) + 2\varphi^2 \lambda$, which has a general solution $\lambda(t) = A_{41} e^{m_{41}t} + A_{42} e^{m_{42}t} + K_{41}$ with $m_{41} = 2\varphi\sqrt{f}$, $m_{42} = -m_{41}$ and $K_{41} = -\frac{[\alpha - (\beta - \gamma)\omega]\psi}{2\varphi^2}$. Also, $y(0) = y_0$ and $\lambda(T) = 0$ gives $A_{41} = \frac{-m_{41}K_{41} + 2fy_0 e^{-m_{41}T}}{m_{41}(e^{m_{41}T} + e^{-m_{41}T})}$ and $A_{42} = -\frac{m_{41}K_{41} + 2fy_0 e^{m_{41}T}}{m_{41}(e^{m_{41}T} + e^{-m_{41}T})}$. Plugging the expressions and simplifying gives $J^m = [\alpha - (\beta - \gamma)\omega] \left\{ \omega T - \frac{[\alpha - (\beta - \gamma)\omega]\psi^2}{4m_{41}\varphi^2} [m_{41}T - \text{th}(m_{41}T)] \right\}$. Deriving J^m with respect to ω and solving for ω gives (17). Plugging $\lambda(t)$ and ω completes the proof. Furthermore, from (17), manufacturer i 's OLNE transfer price, ω_{iol}^{HIVC} , is strictly positive and lower than $\alpha/(\beta - \gamma)$, $i = 1, 2$, which, using (19), implies retailers' non-negative demands. The pollution stock only increases over time, which ensures manufacturers' non-negative net current emissions. \square

Proof of Lemma 1. We first derive the FBNE equilibrium. The Hamilton-Jacobi-Bellman (HJB) equations are respectively:

$$-\dot{V}^m = \text{Max}_{v_i, v_j} \left\{ \sum_i \omega_i (\alpha - \beta p_i + \gamma p_j) - \frac{\sum_i v_i^2}{2} - f y^2 + V_y^m [\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i] \right\}$$

$$-\dot{V}^r = \text{Max}_{p_i} \left\{ \sum_i (p_i - \omega_i) (\alpha - \beta p_i + \gamma p_j) + V_y^r [\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i] \right\}$$

$i = 1, 2, i \neq j$. The maximization on the right-hand side of the HJB equations with respect to v_i and p_i gives $v_i = v_j = -\varphi V_y^m$ and $p_i = \frac{\alpha + (\beta - \gamma)\omega_i - (\beta - \gamma)V_y^r \psi}{2(\beta - \gamma)}$. Since each manufacturer has the symmetric profit structure and $v_i = v_j$, we assume symmetric manufacturers such that $\omega_i = \omega_j = \omega$. We make the following conjectures for the value functions $V^m(y, t) = A(t)y^2 + B(t)y + C(t)$, $V^r(y, t) = D(t)y + E(t)$. To ensure that the HJB equations and the boundary conditions hold for all $y \geq 0$ and all t , after substituting the conjectured value functions into the HJB equations, the following set of equations should be met:

$$\dot{A} = f - 4A^2\varphi^2, \dot{B} = -4AB\varphi^2 + 2A\omega_i(\beta - \gamma)\psi - 2A\psi(\alpha + D(\beta - \gamma)\psi)$$

$$\dot{C} = -\alpha\omega_i + \omega_i^2(\beta - \gamma) - B^2\varphi^2 + B\omega_i(\beta - \gamma)\psi - D\omega_i(\beta - \gamma)\psi - B\psi(\alpha + D(\beta - \gamma)\psi)$$

$$\dot{D} = -4\varphi^2 AD, \dot{E} = -\frac{\alpha^2 + \omega_i^2(\beta - \gamma)^2 + 4BD(\beta - \gamma)\varphi^2 + 2D\omega_i(\beta - \gamma)(-\beta + \gamma)\psi + D^2(\beta - \gamma)^2\psi^2 - 2\alpha(\beta - \gamma)(\omega_i - D\psi)}{2(\beta - \gamma)}$$

with $A(T) = B(T) = C(T) = 0$ and $D(T) = E(T) = 0$. Solving the differential equations and using $J^{mi} = V^m(y_0, 0)$ given that $y_0 = 0$, we obtain the equilibriums satisfying $\omega_{iol}^{HIVC}(t) = \omega_{ifb}^{HIVC}(t)$, $p_{iol}^{HIVC}(t) = p_{ifb}^{HIVC}(t)$, $v_{iol}^{HIVC}(t) = v_{ifb}^{HIVC}(t)$, which completes the proof. \square

Proof of Theorem 5. The objective criteria in the horizontal competition and vertical integration case are:

$$\text{Max}_{p_i, v_i \geq 0} J^i = \int_0^T \left[p_i (\alpha - \beta p_i + \gamma p_j) - \frac{v_i^2}{2} - \frac{f y^2}{2} \right] dt$$

subject to (5), $i = 1, 2$. In an OLNE, the Hamiltonians write:

$$H^i = p_i (\alpha - \beta p_i + \gamma p_j) - \frac{v_i^2}{2} - \frac{f y^2}{2} + \pi_i [\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i]$$

$i = 1, 2, i \neq j$, from which the equations of the costate variables are given by $\dot{\pi}_i = fy$ and the transversality conditions $\pi_i(T) = 0$. Because $\pi_i = \pi_j = \pi$, we can assume symmetric players. Necessary conditions for optimality are respectively $H_{v_i}^i = 0 \Rightarrow v_i = -\varphi \pi$, $H_{p_i}^i = 0 \Rightarrow p_i = [\alpha - (\beta - \gamma)\pi\psi]/(2\beta - \gamma)$. The Hamiltonians are

concave in (v_i, p_i) since the Legendre-Clebsch conditions are satisfied. On the other hand, it can be shown that the necessary conditions are also sufficient for optimality, which implies that the optimal solution is unique. The canonical system then writes $\dot{\pi} = fy$, $\dot{y} = 2\left\{\beta\alpha\psi + [(\beta-\gamma)^2\psi^2 + \varphi^2(2\beta-\gamma)]\pi\right\}/(2\beta-\gamma)$, which yields a

general solution $\pi(t) = A_{21}e^{m_{21}t} + A_{22}e^{m_{22}t} + K_{21}$, with $m_{21} = \sqrt{\frac{2f[(\beta-\gamma)^2\psi^2 + \varphi^2(2\beta-\gamma)]}{2\beta-\gamma}}$, $m_{22} = -m_{21}$ and $K_{21} = -\frac{\alpha\beta\psi}{[(\beta-\gamma)^2\psi^2 + \varphi^2(2\beta-\gamma)]}$. We obtain $A_{21} = A_{22} = \frac{-K_{21}}{(e^{m_{21}T} + e^{-m_{21}T})}$ from $y(0) = 0$ and $\pi(T) = 0$. Plugging the expressions and simplifying completes the proof. Furthermore, from (21), retailer i 's OLNE consumer price, p_{iol}^{TC} , $i = 1, 2$, is strictly positive and lower than $\frac{\alpha}{\beta-\gamma}$, which ensures non-negative retailers' demands. The pollution stock is strictly increasing over time, which guarantees non-negative manufacturers' net emissions. \square

Proof of Theorem 6. Since the game is symmetric, we assume that the optimal value functions are the same for each supply chain, that is, $V^i = V^j = V$. We first construct the Hamilton-Jacobi-Bellman (HJB) equations:

$$-\dot{V} = \text{Max}_{v_i, p_i} \left\{ p_i(\alpha - \beta p_i + \gamma p_j) - \frac{v_i^2}{2} - \frac{fy^2}{2} + V_y[\psi \sum_i (\alpha - \beta p_i + \gamma p_j) - \varphi \sum_i v_i] \right\}$$

$i = 1, 2, i \neq j$. The maximization on the right-hand side of the HJB equations with respect to v_i and p_i gives $v_i = -\varphi V_y$ and $p_i = \{\alpha - (\beta - \gamma)V_y\psi\}/(2\beta - \gamma)$. We make the conjectures for the value functions: $V(y, t) = A(t)y^2 + B(t)y + C(t)$. To ensure that the HJB equations and the boundary conditions hold for all $y \geq 0$ and all t , after substituting the conjectured value functions into the HJB equations, the following set of equations should be met:

$$\begin{aligned} \dot{A} &= -\frac{2Q_1A^2}{(2\beta-\gamma)^2} + \frac{f}{2}, \quad \dot{B} = -\frac{4A}{2(2\beta-\gamma)^2} [\alpha\psi(4\beta^2 - 3\beta\gamma + \gamma^2) + Q_1B] \\ \dot{C} &= -\frac{1}{2(2\beta-\gamma)^2} [2\alpha(4\beta^2 - 3\beta\gamma + \gamma^2)B\psi + Q_1B^2 + 2\alpha^2\beta] \end{aligned}$$

with $A(T) = B(T) = C(T) = 0$, where $Q_1 \equiv [6\beta^3\psi^2 + 2\beta\gamma(5\gamma\psi^2 - 6\varphi^2) - 2\beta^2(7\gamma\psi^2 - 6\varphi^2) + \gamma^2(-2\gamma\psi^2 + 3\varphi^2)]$. Solving the differential equations and plugging the solutions completes the proof. Furthermore, from (25), retailer i 's FBNE consumer price, p_{ifb}^{TC} , is strictly positive. Using (24) in (26), it can be shown that the pollution stock is strictly increasing over time whenever $\beta[6\beta^3\psi^2 + 2\beta\gamma(5\gamma\psi^2 - 6\varphi^2) - 2\beta^2(7\gamma\psi^2 - 6\varphi^2) - \gamma^2(2\gamma\psi^2 - 3\varphi^2)] > (4\beta^2 - 3\beta\gamma + \gamma^2)[(\beta - \gamma)^2\psi^2 + (2\beta - \gamma)\varphi^2]$ is fulfilled. This ensures that manufacturers' net current emissions are non-negative. From (24)-(25), we observe that the abatement efforts and the consumer prices in an FBNE are both increasing with the pollution stock at a decreasing rate. \square

Proof of Theorem 7. After rearranging, we get: $\frac{y_{ol}^{HCVI}(T)}{y^{TI}(T)} = \frac{2\beta}{(2\beta-\gamma)} \cdot \frac{\frac{\text{th}(\vartheta T)}{\vartheta T}}{\frac{\text{th}(\theta T)}{\theta T}}$. Note that $\frac{\text{th}(x)}{x}$ is a decreasing function for $x > 0$ and $\vartheta < \theta$. These lead to $\frac{\text{th}(\vartheta T)}{\vartheta T} > \frac{\text{th}(\theta T)}{\theta T}$. Therefore, $y_{ol}^{HCVI}(T) > y^{TI}(T)$ holds. Let $\beta - \gamma \equiv 2k\varphi^2$ ($k > 0$). Then, $\theta = \delta\sqrt{(k\psi^2 + 1)} > \delta$, and $\frac{y^{TI}(T)}{y^{HIVC}(T)} = \left\{ \frac{[\delta T - \text{th}(\delta T)]}{\delta T} \frac{k\psi^2}{\sqrt{k\psi^2 + 1}} + \frac{2}{\sqrt{k\psi^2 + 1}} \right\} \cdot \frac{\text{th}(\theta T)}{\text{th}(\delta T)}$. Let $\bar{k} \equiv k\psi^2$. Then, for $\bar{k} < 3$, it holds that $\frac{2}{\sqrt{\bar{k}+1}} > 1$. Therefore, for any $0 < \bar{k} < 3$, $\frac{y^{TI}(T)}{y^{HIVC}(T)} > 1$ holds. For $\bar{k} \geq 3$, a sufficient condition for $\frac{y^{TI}(T)}{y^{HIVC}(T)} > 1$ is that $\frac{\delta T - \text{th}(\delta T)}{\delta T} > \frac{\sqrt{\bar{k}+1}-2}{\bar{k}}$. Let $\bar{k}^* = \text{argmax}_{\bar{k} \in [3, \infty)} \left(\frac{\sqrt{\bar{k}+1}-2}{\bar{k}} \right)$, then $\frac{\sqrt{\bar{k}+1}-2}{\bar{k}}|_{\bar{k}^*} = 0.134$ and $\lim_{\bar{k} \rightarrow \infty} \frac{\sqrt{\bar{k}+1}-2}{\bar{k}} = 0$. Also, let $\tau = \delta T$ and, for $\tau > 1$, $\tau^* = \text{argmin}_{\tau \in [1, \infty)} \left(\frac{\delta T - \text{th}(\delta T)}{\delta T} \right)$. Then, $\frac{\delta T - \text{th}(\delta T)}{\delta T}|_{\tau^*} = 0.238$ and $\lim_{\tau \rightarrow \infty} \frac{\delta T - \text{th}(\delta T)}{\delta T} = 1$. Therefore, the sufficient condition $\frac{\delta T - \text{th}(\delta T)}{\delta T} > \frac{\sqrt{\bar{k}+1}-2}{\bar{k}}$ holds for $\bar{k} \geq 3$ and $\tau > 1$, which leads to $\frac{y^{TI}(T)}{y^{HIVC}(T)} > 1$. Lastly, when $\bar{k} \geq 3$ and $0 < \tau \leq 1$, $\frac{y^{TI}(T)}{y^{HIVC}(T)} > 1$ hold if $\frac{[\tau - \text{th}(\tau)]}{\tau} \frac{\bar{k}}{\sqrt{\bar{k}+1}} + \frac{2}{\sqrt{\bar{k}+1}} > \frac{\text{th}(\tau)}{\text{th}(\tau\sqrt{\bar{k}+1})}$. In the case of $\tau = 1$, the latter inequality holds for any $\bar{k} \geq 3$. When $\tau \rightarrow 0$, it holds that $\lim_{\tau \rightarrow 0} \left\{ \frac{[\tau - \text{th}(\tau)]}{\tau} \frac{\bar{k}}{\sqrt{\bar{k}+1}} + \frac{2}{\sqrt{\bar{k}+1}} \right\} = \frac{2}{\sqrt{\bar{k}+1}}$ and $\lim_{\tau \rightarrow 0} \left\{ \frac{\text{th}(\tau)}{\text{th}(\tau\sqrt{\bar{k}+1})} \right\} = \frac{1}{\sqrt{\bar{k}+1}}$. Thus, $\frac{y^{TI}(T)}{y^{HIVC}(T)} > 1$ holds when $\tau < 1$ approaches zero. Also, for all (k, τ) where $\bar{k} \geq 3$ and $0 < \tau \leq 1$, the minimum value of $\left\{ \frac{[\tau - \text{th}(\tau)]}{\tau} \frac{\bar{k}}{\sqrt{\bar{k}+1}} + \frac{2}{\sqrt{\bar{k}+1}} \right\} \cdot \frac{\text{th}(\tau\sqrt{\bar{k}+1})}{\text{th}(\tau)}$ is strictly larger than 1, when numerically computed, confirming that $\frac{y^{TI}(T)}{y^{HIVC}(T)} > 1$. Furthermore, when $\bar{k} \rightarrow \infty$, $\frac{[\tau - \text{th}(\tau)]}{\tau} \frac{\bar{k}}{\sqrt{\bar{k}+1}} + \frac{2}{\sqrt{\bar{k}+1}} > \frac{\text{th}(\tau)}{\text{th}(\tau\sqrt{\bar{k}+1})}$ is satisfied, thereby $\frac{y^{TI}(T)}{y^{HIVC}(T)} > 1$ holds. \square

Proof of Theorem 8. $\frac{y_{ol}^{TC}(T)}{y^{HIVC}(T)} = \frac{\sqrt{2}\beta[\sqrt{2}((\beta-\gamma)\psi^2+4\varphi^2)\rho T-(\beta-\gamma)\psi^2\text{th}(\sqrt{2}\rho T)]}{\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2)-\text{th}(\rho T)]+4\varphi^2\rho T(2\beta-\gamma)} \frac{\text{th}(\rho T)}{\frac{\rho T}{\sqrt{2}\rho T}}$. Since $\frac{\text{th}(\rho T)}{\frac{\rho T}{\sqrt{2}\rho T}} > 1$, a sufficient condition (though not necessary) for $\frac{y_{ol}^{TC}(T)}{y^{HIVC}(T)} > 1$ is $\frac{\sqrt{2}\beta[\sqrt{2}((\beta-\gamma)\psi^2+4\varphi^2)\rho T-(\beta-\gamma)\psi^2\text{th}(\sqrt{2}\rho T)]}{\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2)-\text{th}(\rho T)]+4\varphi^2\rho T(2\beta-\gamma)} > 1 \leftrightarrow \beta^2\psi^2[\rho T \text{sech}(\rho T)^2 + \text{th}(\rho T) - \sqrt{2}\text{th}(\sqrt{2}\rho T)] > \gamma\{\beta\psi^2[\rho T \text{sech}(\rho T)^2 + \text{th}(\rho T) - \sqrt{2}\text{th}(\sqrt{2}\rho T)] - 4\varphi^2\rho T\}$. Since $\rho T \text{sech}(\rho T)^2 + \text{th}(\rho T) - \sqrt{2}\text{th}(\sqrt{2}\rho T) < 0$, the sufficient condition for $\frac{y_{ol}^{TC}(T)}{y^{HIVC}(T)} > 1$ is $\gamma > \bar{\gamma} = \frac{\beta^2\psi^2[\sqrt{2}\text{th}(\sqrt{2}\rho T)-\rho T \text{sech}(\rho T)^2-\text{th}(\rho T)]}{\beta\psi^2[\sqrt{2}\text{th}(\sqrt{2}\rho T)-\rho T \text{sech}(\rho T)^2-\text{th}(\rho T)]+4\varphi^2\rho T} > 0$, where $\rho = \varphi\sqrt{2f}$. \square

Proof of Theorem 9. $\frac{y^{HCVI}(T)}{y^{TC}(T)} = \frac{\beta(\beta-\gamma)\psi^2[\varphi T\sqrt{2f}(2-\text{sech}(\varphi T\sqrt{2f})^2)-\text{th}(\varphi T\sqrt{2f})]+4\varphi^3T\sqrt{2f}(2\beta-\gamma)}{2\sqrt{2}\varphi^2T\sqrt{f(2\beta-\gamma)}[(\beta-\gamma)^2\psi^2+\varphi^2(2\beta-\gamma)]} \frac{\text{th}(\vartheta T)}{\text{th}(\varphi\sqrt{2f}T)}$. A sufficient condition (though not necessary) for $\frac{y^{TC}(T)}{y^{HIVC}(T)} > 1$ is $\frac{\beta(\beta-\gamma)\psi^2[\varphi T\sqrt{2f}(2-\text{sech}(\varphi T\sqrt{2f})^2)-\text{th}(\varphi T\sqrt{2f})]+4\varphi^3T\sqrt{2f}(2\beta-\gamma)}{2\sqrt{2}\varphi^2T\sqrt{f(2\beta-\gamma)}[(\beta-\gamma)^2\psi^2+\varphi^2(2\beta-\gamma)]} > 1 \leftrightarrow \beta(\beta-\gamma)\psi^2[\varphi T\sqrt{2f}(2-\text{sech}(\varphi T\sqrt{2f})^2) - \text{th}(\varphi T\sqrt{2f})] + 4\varphi^3T\sqrt{2f}(2\beta-\gamma) - 2\sqrt{2}\varphi^2T\sqrt{f(2\beta-\gamma)}[(\beta-\gamma)^2\psi^2 + \varphi^2(2\beta-\gamma)] > 0$ as $\frac{\text{th}(\vartheta T)}{\text{th}(\varphi\sqrt{2f}T)} > 1$. Because $[\varphi T\sqrt{2f}(2-\text{sech}(\varphi T\sqrt{2f})^2) - \text{th}(\varphi T\sqrt{2f})] > 0$, the above condition is satisfied if $\varphi > \bar{\varphi} = \frac{(\beta-\gamma)\psi}{\sqrt{3(2\beta-\gamma)}}$. \square

Proof of Theorem 10. $\frac{y^{TI}(T)}{y_{ol}^{TC}(T)} = \frac{\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2)-\text{th}(\rho T)]+4\varphi^2\rho(2\beta-\gamma)T}{4\beta\theta\varphi^2T} \frac{\text{th}(\theta T)}{\text{th}(\rho T)}$ and $\frac{\text{th}(\theta T)}{\text{th}(\rho T)} > 1$. A sufficient condition (though not necessary) for $\frac{y^{TI}(T)}{y_{ol}^{TC}(T)} > 1$ is $\beta(\beta-\gamma)\psi^2[\rho T(2-\text{sech}(\rho T)^2) - \text{th}(\rho T)] + 4\varphi^2\rho(2\beta-\gamma)T > 4\beta\theta\varphi^2T$. As $[\rho T(2-\text{sech}(\rho T)^2) - \text{th}(\rho T)] > 0$, which holds if $\varphi > \tilde{\varphi} = \beta\psi\sqrt{\frac{\beta-\gamma}{(2\beta-\gamma)^2-2\beta^2}}$ for $\beta > \tilde{\beta} = \frac{\gamma}{(2-\sqrt{2})}$. \square