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Explaining the Ammunition Shortage: The Show vs. Have Military Power Game

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Abstract

The return of high-intensity conflict at Europe's borders has exposed the massive under-investment in ammunition by many Western states, relative to the normal availability of main weapons. This paper complements existing explanations for this relative imbalance with insights from a signaling game. In an environment of uncertain information, where neither military budgets nor investment in ammunition can be observed, low-budget states may imitate the procurement strategies of high-budget states to project an image of power. In this context, high-budget states have an incentive to over-invest in main weapons and neglect ammunition as a strategy to dissuade imitation by low-budget states. While this behavior may be rational in times of peace when perceptions may matter as much as capabilities, during wartime, the deficit in ammunition is extremely harmful, as it increases the likelihood of the opponent's victory.

Keywords - High-intensity warfare, Signaling game, Military power, Perceived power.

JEL Classification - D82; F52; H56; H57.

1 Introduction

Since the fall of the Iron Curtain, Western policymakers, particularly the European ones, have lived under the illusion of the ‘peace dividend.’ As the Soviet Union disappeared and the Russian economy went through a deep crisis, with its armed forces weakened during the transition to a market-based economy, Western policymakers championed a reduction in defense spending and redeployed the spared resources toward civil and social programs. According to this rationale, shared by many laypeople, after the end of the communist-capitalist confrontation, Western countries would face no major threats in the future. In particular, autocratic regimes were expected to play the market game, get wealthier and better educated, and finally embrace democratic values (Fukuyama, 1992).

Moscow’s full-scale invasion of Ukraine in February 2022 ended this illusion. The aggression against a pro-European country at the EU’s border revealed that symmetrical, land-based, high-intensity warfare is not a thing of the past. Despite early warnings (the invasion of Crimea in 2014), the war in Ukraine shook the established military doctrines of EU member countries, which overnight had to confront their salient lack of preparedness for this type of conflict, a conclusion that underlies the emergency military guidelines wrote out in the Strategic Compass of March 2022 (Hartley, 2023)¹. In July 2024, an influential bi-partisan commission appointed by the US Congress to review the US National Defense Strategy of 2022 underlined the significant unpreparedness of the US military forces to fulfill its missions (USCNSD, 2024).

The continuing war in Ukraine revealed some basic lessons from the past, which had been somehow forgotten. Besides high-tech sophisticated and quite expensive weapon systems, a symmetric high-intensity land-based war requires massive artillery, huge stocks of ammunition, basic equipment, lots of spare parts, workshops to repair damaged materials, and substantial logistics, in a war universe closer to World War II than Star Wars.

To date, the most representative dimension of the West’s unpreparedness for a high-intensity war has been observed in the domain of ammunition. The journal *Le Monde*, on February 18, 2023, cited a French Parliament report, according to which the French Army’s ammunition stock would not last for more than a few weeks in a high-intensity conflict.² War simulations performed in 2022 showed that, in a high-intensity conflict, the United Kingdom would exhaust its ammunition stocks in just eight days (HouseofCommons, 2024), while the Bundeswehr’s stocks would last somewhere between a few hours and a few days Aries et al. (2023).

In an interview for the *Financial Times* on July 1st, 2024 - Rob Johnson, former director of the office of net assessment and challenge with the UK Ministry of Defence acknowledged that “In any larger-scale operation, we would run out of ammunition rapidly... Our defences are too thin, and we are not prepared to fight and win an armed conflict of any scale”. Retired Air Marshal Edward Stinger, told the *Financial Times* that Britain had a “Shopfront military, one that can excel on display and exercises... but when you go beyond the facade, there’s precious little on the shelves and no production lines behind it”.³

While many European governments revealed extreme lack of preparedness in fighting a high-intensity, land-based war, the United States are not exempt of similar criticism, despite its substantial military strength. In a report on January 2023, Marc Cancian, a senior advisor at the Center for Strategic and International Studies called attention on the shortage of 155mm artillery rounds to which the US army is exposed after delivering substantial quantities to Ukraine.⁴ In an interview for the journal *National Security* on July 18, 2023, the same expert argues that NATO forces could run out some varieties of ammunition if there is a war with China or some other major power. In 2023, the US had to buy ammunition from South Korea, as its production line could not follow the consumption of shells in Ukraine. Beaver and Fein (2024) note that the poor ammunition planning by the US Department of defense is a trait of the post-1989 period. They underline that “The Pentagon was caught off guard by the amount of artillery shells needed by the Ukrainian and Israeli military after being attacked, and supply is short.” They call attention on the fact that, by mid-2024, the U.S. air force including 111 bombers could launch no more than 4.95 joint air-to-surface standoff missiles per bomber, and the Navy’s 23 active destroyers, cruisers, and submarines could fire zero Tomahawks and at most 1.76 standard missile each. Finally,

¹In an official EU document, “European Defence Industrial Strategy of 2024”, the European Commission acknowledged that between 2006 and 2020, EU member countries recorded a deficit of 1.100 billion euros in military spending, estimated against the NATO spending norm of 2% of GDP.

²See “L’armée française manque de munitions pour la haute intensité”, *Le Monde*, February 18, 2023

³See “UK military unprepared for ‘conflict of any scale’, warns ex-defence official”, in the *Financial Times*, July 1st, 2024.

⁴See “Rebuilding U.S. Inventories: Six Critical Systems”, Center for Strategic and International Studies, January 9, 2023; <https://www.csis.org/analysis/rebuilding-us-inventories-six-critical-systems>

in July 2024, the above-mentioned Report of the Commission to review the US National Defense Strategy bluntly states “Previous NDS Commissions have warned that Department of Defense has systematically under-invested in munitions, choosing to raid these accounts as quick fixes to solve budget shortfalls” (USCNSD, 2024). The report further emphasizes that “The Ukraine war and the Israel-Hamas war, however, vividly demonstrate that modern wars are likely to be protracted and consume a lot of munitions, from the relatively basic 155-mm artillery rounds, to Army Tactical Missile System (ATACMS) missiles, to air defense interceptors, and *the United States simply does not have enough of such munitions on hand*”.

In general, the peace dividend rationale and the resulting budget cuts provide the main explanation for the poor preparedness for large conflicts of the Western states. Other explanations are specific to the EU: they include policy and defense industrial base fragmentation, excessive rivalry between EU states focused on narrow national interests, strong competition between EU defense firms (Hartley, 2023), reluctance by private and public funding organisations to support the defense sector⁵, conflicts between various EU institutions (Csernaton, 2021), and the limited coordination role of the European Commission (Besch, 2019).

However, none of these useful explanations can account for the notorious shortage of ammunition *relative* to the stock of major weapons. Following the pioneering analysis by Veblen (1899), a substantial body of literature in social sciences has been dedicated to the analysis of conspicuous or positional goods. Veblen indicated that the consumption of luxury goods can be motivated by humans’ desire to signal their wealth and thus obtain a higher status. Psychologists have been interested in identifying the emotional drivers of this behavior, such as pride, self-image, social image, desire for uniqueness, and so on (see Kumar et al. (2022) for a meta-analysis). These psychological motives might have also contributed to the over-investment in fancy, high-tech weapons, which can be viewed as positional goods in a state’s arsenal.

In this paper, we provide an original explanation to this relative shortage of ammunition, spare parts and other invisible inputs, and the mirror over-investment in fancy weapons, building on traditional signaling game literature.⁶ In contrast to the literature in psychology, our analysis emphasizes the weapon procurement strategy as a means of conveying information about the true military power of a state, in a context where external observers have only imperfect information about this strategic dimension. Limited information is an essential feature of international defense policy, where states try to emphasize their strengths and hide their weaknesses.

National defense, like any form of defense, is dissuasive if the opponent believes that you are stronger than him or her. In times of limited conflict or peace, the *swaggering goal* of military force receives substantial weight. As defined by Art (1980), swaggering is expressed in general by “displaying one’s military might at military exercises and national demonstrations and buying or building the era’s most prestigious weapons”. While the major weapons can be displayed in any demonstration of power, the stock of ammunition (and spare parts) is private information. Imagine now that the budget dedicated to conventional weapon procurement is private information, and that states have either high or low budgets. In this case, a low-budget state might buy and show as many main weapons as a high-budget state, to the expense of the stock of ammunition, just to manipulate the beliefs of the external observers and make them believe that it has a large budget, which in turn would prompt them to avoid conflict. Following the classical paper by Rosen (1986), many scholars have studied games in which players invest in signals aiming at deceiving their opponents, or ‘bluffing’ as referred to in the poker game. For instance, individuals can send deceiving messages to prompt their opponents into thinking that they are stronger (or wealthier) than they really are in dynamic auctions (Avery, 1998) or in the workplace when compensation is based on the outcome of a tournament (Ederer, 2010).

In our context, this imitation strategy cannot be an equilibrium, since high-budget states might increase the procurement of main weapons up to the point where the imitation strategy no longer pays for a low-budget state. This mechanism is present in the analysis by Spence (2002), where high-skilled workers would push their education years to the limit where low-skilled workers, with a higher cost per education year, obtain no benefit from imitating them.⁷

The game features a non-trivial separating equilibrium in which high-budget states under-invest in ammunition, spare parts, and other low-tech resources, to the benefit of visible, fancy high-tech weapons systems. The results of our analysis show that over-investment in visible systems

⁵The European Investment Bank has denied support to defense projects until May 2024

⁶Signaling games represent a class of games with significant applications to economic issues (see the surveys by Riley (2001) and Sobel (2020)). They are also widespread in the management literature Cowen (2011).

⁷See also Vickers (1986) and Gal-Or (1989) for a similar reasoning as applied to monetary policy credibility and warranty coverage. Clingingsmith and Sheremeta (2018) built a lab experiment and find out that the results are consistent with a signaling equilibrium.

essentially depends on the budget gap between states and a ‘show-off’ parameter that adjusts the weight of perceived military strength against actual military strength in the state’s objective function. It is highly probable that in times of peace, states have a stronger preference for conveying an image of military strength, even if they must sacrifice some essential inputs for their actual defense. However, in times of conflict, the costs of showing off in good times appear to be a serious burden.

A related application of the signaling model to defense economics was developed by [Lipow and Feinerman \(2001\)](#), who analyzed factors explaining the relative under-investment in the quality of Israeli troops compared to the relatively high quality of their weapons. In their model, the quality of commanding officers cannot be observed directly. A good officer can make a more efficient use of the resources under his or her control, which can be shared between improving the qualification of the troops, or buying equipment. Investment in troops is an unobserved variable. Without a signaling strategy, low-skilled officers would invest in weapons as much as the more efficient high-skilled officers. The latter can signal their type by over-investing in weapon quality, which prompts lesser-quality officers to choose their optimal bundle of troop training and weapon acquisition.

The paper is organized as follows. The next section introduces the main assumptions of the model. In Section 2, we analyze optimal choices under perfect information. The imperfect information case is analyzed in Section 3, which also introduces the signaling equilibrium. Section 4 is our conclusion.

2 The model

2.1 Technology

Many weapon systems are made up of relatively expensive devices capable of launching a lethal projectiles toward enemy targets and various incoming threats. In this category of weapons, we find traditional guns and their ammunition (bullets), traditional howitzers and their artillery rounds, and more modern launchers and their rockets or missiles, such as those fired by the man-portable, fire-and-forget Stinger and Javelin, or the more sophisticated HIMARS, which have proven their large effectiveness in Ukraine ([Petraeus and Roberts, 2024](#)). In these examples, launchers and ammunition complement each other in the projection of force. A similar complementarity logic applies to any type of weapon, since ultimately, all weapons require a significant amount of ammunition to be effective, whether it be a howitzer, a lethal drone, or the most advanced attack helicopter. Similar to the weapon and ammunition complementarity, one can consider the weapon and spare parts association; in a land-based, high-intensity war, weapons often break in contact with the enemy. Their proper functioning requires a substantial maintenance effort and large inventories of spare parts.

This complementarity extends to many other aspects of the production of military power, which require relatively fixed proportions to achieve the greatest impact. Communications, troops, logistics, satellite surveillance, weapons of various kinds, and their ammunition must work together. At a more aggregate level, substitutions can be made to some extent, but no one can imagine a modern high-intensity war being won solely by naval forces or troops alone. Thus, complementarity is the key property of any meaningful military production function. In this paper, we assume that the functioning of a weapon system in the battlefield involves *perfect complementarity* between launchers and ammunition.

Furthermore, to use an intuitive notation, we will refer to launchers as *howitzers*, in quantity C and to ammunition as *shells*, in quantity S . We assume that in a standard land-based conflict a howitzer can shoot α shells before it breaks down. Thus, any efficient allocation (or kit) of howitzers and shells should verify:

$$S = \alpha C. \tag{1}$$

The bold line in [Figure 1](#) represents these efficient bundles. For any given amount of howitzers C , disposing of more or less than αC shells is tantamount to a waste of resources. Investing more in shells is useless, since the excess shells cannot be used; investing less, means that some howitzers cannot be used.

As the two inputs are perfect complements, the firing power of the system, F , can be represented by the elementary fixed-proportion production function:

$$F = \min\left\{C, \frac{S}{\alpha}\right\}. \tag{2}$$

According to this expression, the firing power is determined by the smallest of the number of howitzers and the adjusted number of shells.

Given the fixed-proportion technology, the distance D between the origin and any of the efficient sets (C, S) on the bold line in Figure 1 can be interpreted a measure of the *conventional military power* of a state :

$$D = \sqrt{S^2 + C^2}. \quad (3)$$

This measure allows us to capture through a single-dimensional variable the contribution of the two complementary factors. For combinations that are not on the efficient line, the distance will reflect the firing power as determined by the binding factor (i.e., the factor which is in short supply, ammunition in our context).

2.2 Main goal of the state

In line with the four-goal analysis by Art (1980), we assume that states care about their actual military power as dedicated to the goals of defense, deterrence and compellence, and also obtain a benefit if they manage to persuade external observers that they dispose of a substantial military power, what was referred to as swaggering in Art (1980). Actual and perceived power are related, but, as our analysis will point out, are not necessarily identical in an imperfect information setting.

This dual goal can be captured in a simple way by a linear objective function of the state i :

$$V_i = (1 - \gamma)D_i + \gamma E[D_i|I], \text{ with } \gamma \in (0, 1) \text{ and } i = \{L, H\}. \quad (4)$$

where D_i stands for actual military power as defined in Equation (3), and $E[D_i|I]$ is the perceived military power by external observers, contingent on their information set $\{I\}$, which includes all the relevant observed variables. The 'show-off' parameter γ captures policymakers' preferences as it tunes the weight of the two partial goals in the general goal.

In times of limited conflict or peace, the relative importance of image can be quite large, while it should be low in conflict times, when the show-off rationale is replaced by a must-win rationale.

2.3 The budget constraint

For any state i , the conventional weapon budget can be used to buy howitzers (C_i) and shells (S_i). Let p^C be the price of howitzers and p^S the price of shells. The budget constraint of a state i can be written as:

$$p^C C_i + p^S S_i \leq B_i \text{ with } i = (H, L). \quad (5)$$

If we assume that the cost and price of a shell, which requires relatively basic technology to be produced, does not vary from one country to another, we can use it as numeraire and set $p^S = 1$.⁸ We denote the relative price $\eta = p^C/p^S > 1$. Then the budget constraint of the state can be written as:

$$\eta C_i + S_i \leq B_i. \quad (6)$$

However, given the former objective function in which both actual and displayed military power require the full use of inputs, it is certain that the budget constraint is saturated. We therefore can write the budget constraint simply as:

$$B_i = \eta C_i + S_i. \quad (7)$$

We denote by \bar{C}_i the maximal amount of howitzers a state can buy (the entire budget is used to buy howitzers), so $\bar{C}_i = B_i/\eta$.

2.4 Sources of heterogeneity

States differ with respect to many elements of their military power. In this analysis, on a Occam razor principle logic, we will consider that countries are almost identical, with heterogeneity measured along only one dimension of military power: the size of the budget dedicated to procurement

⁸This is obviously a simplification. The NATO standard 155-mm artillery rounds vary with respect to their explosive power and distance range; by mid-2024 their market price vary between 4000 and 10000 USD a piece.

of conventional weapons per million inhabitant.⁹ Governments use to disclose the total defense related expenses, and research institutions such as the SIPRI, compile international comparisons of military expenses, including wages and pensions of the military, expenses on nuclear defence, logistics and maintenance, R&D, and expenses for conventional weapons. However, many governments would conceal the amount of expenses dedicated to the procurement of conventional weapons, which is considered as strategic information. This intrinsic opacity creates a typical imperfect information environment, where high and low budget states coexist.¹⁰

For our analysis, we assume that a fraction λ of the states (*type* – H states, or simply H states) have a high budget B_H , and a fraction $(1 - \lambda)$ of the states (*type* – L states or L states) have a low budget B_L , with $B_H > B_L$. External observers, including the military intelligence of rival states, do not know the exact budget of other states; however, they know the distribution of states by budget (λ), and might use observed signals to infer the type of the state.

3 The perfect information case

In the context of military power and military expenses, assuming perfect information is not realistic. However, it is useful to address this as a baseline against which we can later analyze the more plausible case of imperfect information.

Under perfect information $E[D_i|I] = D_i$, thus a state i that seeks to maximize V_i will choose the efficient kit (C, S) ($S = \alpha C$, according to Equation 1) that satisfies the budget constraint:

$$B_i = \eta C_i + \alpha C_i = (\eta + \alpha) C_i. \quad (8)$$

The perfect information, optimal amount of howitzers and shells is:

$$C_i^* = \frac{1}{\eta + \alpha} B_i \quad (9)$$

$$S_i^* = \frac{\alpha}{\eta + \alpha} B_i. \quad (10)$$

For any two countries, one of the type H and the other of the type L ($B_H > B_L$), it turns out that $C_H^* > C_L^*$ and $S_H^* > S_L^*$.

The perfect information (conventional) military power is:

$$D_i^* = \sqrt{S_i^{*2} + C_i^{*2}} = \sqrt{(1 + \alpha^2) C_i^{*2}} = B_i \sqrt{\frac{(1 + \alpha^2)}{(\eta + \alpha)^2}} \quad (11)$$

or, alternatively,

$$D_i^* = \sqrt{(B_i - \eta C_i^*)^2 + \left(\frac{1}{\alpha} (B_i - \eta C_i^*)\right)^2} = (B_i - \eta C_i^*) \sqrt{\frac{(1 + \alpha^2)}{\alpha^2}} \text{ with } C_i^* = \frac{1}{(\eta + \alpha)} B_i \quad (12)$$

We represent in Figure 1 the technology efficient line (Equation 1) and the budget lines for a state H and for a state L (Equation 7). The slope of the efficient line is $1/\alpha$ and the slope of the budget line is $-1/\eta$. Also, the optimal choices are represented as points M and N . Thus, the distance OM (and ON) correspond to D_L^* (and D_H^* , respectively).

4 The imperfect information case

In an imperfect information setting, the military budget (for conventional weapons) of any state cannot be observed; however, the frequency λ of H states in the total population of states is known. With respect to the inputs of the production function, we assume that only the amount of main weapons (howitzers, launchers, etc.) can be observed through the orders placed by the Ministry of defense to main contractors, satellite images that allow them to be counted in the field, and other intelligence gathering methods. On the other hand, the amount of ‘low-tech’ inputs (shells, spare parts, ammunition of various kinds) cannot be observed by external observers.

⁹Alternatively, instead of considering the budget per person, the analysis would compare only countries of a similar size.

¹⁰Adding additional heterogeneity, such as a different relative price η , would not change in a significant way the structure of the problem.

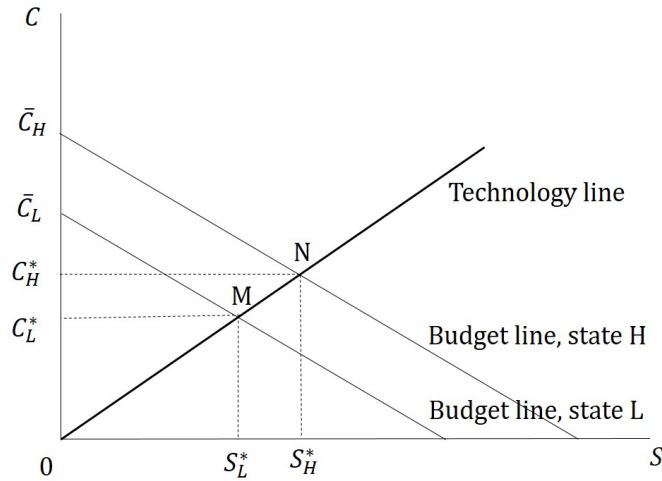


Figure 1: Optimal choices under perfect information.

4.1 No trivial separating equilibrium

By 'trivial' separating equilibrium we understand a situation in which each type of state buys, in the imperfect information setting, its perfect information optimal kit, (C_H^*, S_H^*) and (C_L^*, S_L^*) respectively. If such an equilibrium exists, observers' equilibrium beliefs must be correct:

$$\Pr[\text{type}_L | C = C_L^*] = 1 \quad (13)$$

$$\Pr[\text{type}_H | C = C_H^*] = 1 \quad (14)$$

Proposition 1 *Under imperfect information, if γ is large enough ($\gamma > \frac{\eta}{\eta+\alpha}$), the efficient choices under perfect information, (C_H^*, S_H^*) for state H and (C_L^*, S_L^*) for state L, cannot be an equilibrium of this game. On the other hand, if $\gamma < \frac{\eta}{\eta+\alpha}$ separation prevails, the two bundles can be an equilibrium.*

Proof 1 *A deceptive state L aiming to manipulate observers' beliefs, would acquire as many howitzers as a state H, that is $C_L = C_H^* > C_L^*$ (and consequently it would undercut the procurement of shells to $S_L = B_L - \eta C_H^*$). By so doing, it makes observers believe it is of strong (type H), true, at the expense of a lower military power. The latter falls to $\tilde{D}_L(C_H^*)$ as defined by:*

$$\tilde{D}_L(C_H^*) = (B_L - \eta C_H^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} < D_L^* \quad (15)$$

It is interesting for a deceptive L state to adopt the imitation strategy if:

$$(1 - \gamma)\tilde{D}_L(C_H^*) + \gamma E[D | C_H^*] > (1 - \gamma)D_L^* + \gamma E[D | C_L^*] \quad (16)$$

This is equivalent to:

$$\begin{aligned} (1 - \gamma)(B_L - \eta C_H^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} + \gamma(B_H - \eta C_H^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} &> (B_L - \eta C_L^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} \\ \gamma[(B_H - \eta C_H^*) - (B_L - \eta C_H^*)] &> \eta(C_H^* - C_L^*) \\ \gamma &> \frac{\eta}{\eta + \alpha} \end{aligned} \quad (17)$$

If $\gamma < \frac{\eta}{\eta+\alpha}$, a 'trivial' separating equilibrium exists. In this situation, the state has set the effective development of military power as its priority goal, and pays only limited interest to the image of strength it projects to outsiders. If $\gamma > \frac{\eta}{\eta+\alpha}$, states assign substantial weight to the image they convey to outside world. In this case, there is an incentive for L states to copy the howitzer procurement strategy of the H states, thus the 'trivial' separating equilibrium in which both states acquire the perfect information, optimal kit of howitzers and shells, does not exist.

In the following we focus on the most interesting case where $\gamma > \frac{\eta}{\eta+\alpha}$. In this case, at least some L states if not all of them, have an incentive to deviate and imitate the howitzer procurement strategy of the H states, to make observers believe that they are of the strong type.

4.2 A naive hybrid equilibrium

We thus study a hybrid equilibrium in which some L states choose to imitate the H states and buy as many howitzers as the latter ($C_L = C_H^*$). Let us denote by μ the fraction of L states that adopt the imitation strategy.

In this subsection, we assume that the H state cannot adjust the amount of howitzers it buys (it is subject to procurement stickiness), which explains why we refer to this equilibrium as ‘naive’. In the next sub-section, the analysis is generalized by allowing H states (and not only L states) to ‘strategically’ adjust the amount of howitzers bought.

Recall that if a H state buys the perfect information amount C_H^* , the number of shells is $S_H^* = B_H - \eta C_H^*$ and the associated military power is:

$$D_H^* = (B_H - \eta C_H^*) \sqrt{\frac{(1 + \alpha^2)}{\alpha^2}}. \quad (18)$$

A deceptive state L which buys $\tilde{C}_L = C_H^*$ and $\tilde{S}_L = B_L - \eta C_H^*$ obtains a lower military power compared to the perfect information case (the tilde indicates a suboptimal choice):

$$\tilde{D}_L(C_H^*) = (B_L - \eta C_H^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} < D_L^*. \quad (19)$$

We represent this situation in Figure 2. The distance OM' corresponds to the military power of the deceptive state L .

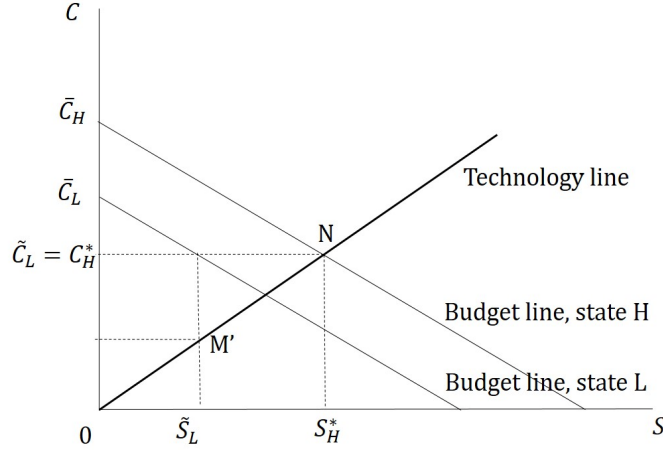


Figure 2: The ‘naive’ hybrid equilibrium.

For an external observer, the howitzer procurement strategy of a state can be written as:

$$\begin{cases} \sigma(H) = (C_H^*, S_H^*) \\ \sigma(L) = \begin{cases} (\tilde{C}_L = C_H^*, \tilde{S}_L = B_L - \eta C_H^*) & \text{with probability } \mu \\ (C_L^*, S_L^*) & \text{with probability } (1 - \mu) \end{cases} \end{cases}$$

We use Bayes’ rule to determine the equilibrium beliefs:

$$\begin{aligned} \Pr[\sigma(H)|C_H^*] &= \frac{\Pr[C_H^*|\sigma(H)] \Pr(\sigma(H))}{\Pr[C_H^*|\sigma(H)] \Pr(\sigma(H)) + \Pr[C_H^*|\sigma(L)] \Pr(\sigma(L))} = \frac{\lambda}{\lambda + \mu(1 - \lambda)} \\ \Pr[\sigma(H)|C_L^*] &= 0 \end{aligned}$$

leading to the expected military power conditional upon the number of howitzers $E[D|C_i]$:

$$E[D|C_L^*] = D_L^* = (B_L - \eta C_L^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} \quad (20)$$

$$\begin{aligned} E[D|C_H^*] &= \frac{\lambda}{\lambda + \mu(1 - \lambda)} D_H^* + \frac{\mu(1 - \lambda)}{\lambda + \mu(1 - \lambda)} \tilde{D}_L(C_H^*) \\ &= \frac{\lambda}{\lambda + \mu(1 - \lambda)} (B_H - \eta C_H^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} + \frac{\mu(1 - \lambda)}{\lambda + \mu(1 - \lambda)} (B_L - \eta C_H^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} \\ &= \left[(B_H - \eta C_H^*) - \frac{\mu(1 - \lambda)}{\lambda + \mu(1 - \lambda)} (B_H - B_L) \right] \sqrt{\frac{1 + \alpha^2}{\alpha^2}} \end{aligned} \quad (21)$$

We recall that λ is the frequency of H states; the higher λ and the lower μ , the higher $E[D|C_H^*]$ is.

In this naive hybrid equilibrium, L states should be indifferent between copying the strategy of the H state, or following their perfect information investment in howitzers (that signals their type). This indifference condition allows us to determine the equilibrium frequency of cheaters, μ .

$$\begin{aligned} V_L(C_L^*) &= V_L(C_H^*) \\ (1 - \gamma)D_L^* + \gamma E[D|C_L^*] &= (1 - \gamma)\tilde{D}_L(C_H^*) + \gamma E[D|C_H^*] \\ (B_L - \eta C_L^*) &= (1 - \gamma)(B_L - \eta C_H^*) + \gamma \left[(B_H - \eta C_H^*) - \frac{\mu(1 - \lambda)}{\lambda + \mu(1 - \lambda)} (B_H - B_L) \right] \\ \mu &= \frac{\lambda}{(1 - \lambda)} \left[\frac{\gamma(B_H - B_L)}{\eta(C_H^* - C_L^*)} - 1 \right] \end{aligned} \quad (22)$$

A hybrid equilibrium is possible for $\mu \in (0, 1)$. It can be easily shown that condition (17) ensures that $\mu > 0$. At least some states L will always imitate the H states (if the latter implement the procurement strategy C_H^*). The condition $\mu < 1$ involves that $\gamma < \frac{1}{\lambda} \frac{\eta}{\eta + \alpha}$. From these two inequalities we infer that a hybrid equilibrium exists for:

$$\frac{\eta}{\eta + \alpha} < \gamma < \frac{1}{\lambda} \frac{\eta}{\eta + \alpha}. \quad (23)$$

As shown in Proposition 1, for $\gamma < \frac{1}{\lambda} \frac{\eta}{\eta + \alpha}$ the trivial separating equilibrium prevails. For $\gamma > \frac{1}{\lambda} \frac{\eta}{\eta + \alpha}$ the equilibrium is of the pooling type ($\mu = 1$), all L states copy the howitzer procurement strategy of the H states.

However, the pooling equilibrium cannot exist if the frequency of H states is relatively small. Indeed, if $\lambda < \frac{\eta}{\eta + \alpha}$ then $\frac{1}{\lambda} \frac{\eta}{\eta + \alpha} > 1$. Since $\gamma < 1$, it turns out that $\gamma < \frac{1}{\lambda} \frac{\eta}{\eta + \alpha}$.

4.3 The arms race and the signaling equilibrium

In a pooling or hybrid equilibrium, H states are penalized, as external observers cannot precisely infer their military strength from observation of their howitzer procurement strategy. For $\mu \in (0, 1)$ and $\lambda \in (0, 1)$, the expected military power as defined in Equation (21) is lower than the perfect information level: $E[D|C_H^*] < D_H^*$.

If a state deviates and buys (slightly) more howitzers than all the others, external observers should interpret this move as a signal for a H -type state.¹¹ In this context, a state H has an incentive to buy slightly more howitzers than the others, because an incremental increase in the amount of howitzers has a negligible impact on the actual military power of this state, while bringing about a quantum jump in the *perceived* military strength of this state (from $E[D|C_H^*]$ to D_H^*).

However, as long as the military budget of the L states is not exhausted, any incremental increase in the procurement of howitzers can be copied by L states, leading to a new hybrid equilibrium. This dynamics is tantamount to a race to the top for the procurement of visible weapons.

When H states increase the level of investment in howitzers to C_H^s (with $C_H^s > C_H^*$) for signaling purpose, and a fraction μ of the L states imitate them, Equation (22) simply becomes:

¹¹Indeed, the opposite beliefs cannot be an equilibrium of this game: a L state has no incentive to raise its investment in howitzers and sacrifice its military power just to signal itself since $D_L^* < E[D|C_L^*]$.

$$\mu(C_H^s) = \frac{\lambda}{(1-\lambda)} \left[\frac{\gamma (B_H - B_L)}{\eta (C_H^s - C_L^*)} - 1 \right]. \quad (24)$$

Obviously, the equilibrium frequency of imitators among the L states declines with C_H^s :

$$\frac{\partial \mu(C_H^s)}{\partial C_H^s} < 0. \quad (25)$$

The over-investment in howitzers by a state H contributes to reducing both its military power and that of its potential imitator. By increasing C_H , the high-budget state increases the cost of imitation and reduces the gain in reputation for the low-budget state. Imitation ceases to be effective when C_H is high enough for the cost of replicating the signal (displaying the same high amount of howitzers) to exceed the gain in reputation for the imitator. Formally, imitation cancels-out as a meaningful strategy of the low-budget state for a critical \hat{C}_H , defined by $\mu(\hat{C}_H) = 0$ or:

$$\hat{C}_H = C_L^* + \frac{\gamma}{\eta} (B_H - B_L). \quad (26)$$

By adopting this high level of investment in howitzers, the state H unambiguously signals its type, since potential imitators have no further incentive to copy it. The resulting signaling equilibrium grants separation of types.

Figure 3 represents this signaling equilibrium in which H states over-invest in howitzers as compared to the perfect information optimum, and relatively neglect the stock of ammunition.

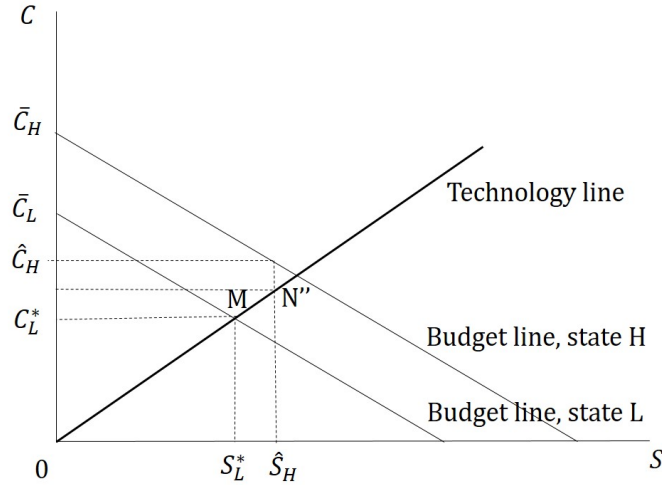


Figure 3: Signaling equilibrium.

Proposition 2 *In the signaling equilibrium, the over-investment in howitzers by the H -state is increasing in the ‘show-off’ parameter γ , in the budget gap $(B_H - B_L)$ and the technology coefficient α ; it decreases in the relative price of howitzers over shells, η .*

Proof 2 *The over-investment in howitzers by the H -state is:*

$$\begin{aligned} \hat{C}_H - C_H^* &= C_L^* - C_H^* + \frac{\gamma}{\eta} (B_H - B_L) \\ &= \frac{1}{\eta + \alpha} (B_L - B_H) + \frac{\gamma}{\eta} (B_H - B_L) \\ &= (B_H - B_L) \left[\frac{\gamma}{\eta} - \frac{1}{\eta + \alpha} \right]. \end{aligned} \quad (27)$$

Signs of the partial derivatives of the over-investment term with respect to parameters are obvious. The sign of the partial derivative with respect η is the sign of the term $[\eta - (\eta + \alpha) \sqrt{\gamma}]$, which is negative for $\gamma > \frac{\eta}{\eta + \alpha}$ (which is the existence condition of the hybrid equilibrium).

The imitation-proof military power of the H state (the distance ON'' in Figure 3) is:

$$\tilde{D}_H(\hat{C}_H) = (B_H - \eta\hat{C}_H) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} \quad (28)$$

$$= [(1 - \gamma)B_H + \gamma B_L - \eta C_L^*] \sqrt{\frac{1 + \alpha^2}{\alpha^2}}. \quad (29)$$

On the other hand, in this equilibrium L states implement their perfect information optimal choice (S_L^*, C_L^*) and obtain the defense level D_L^* , previously defined as:

$$D_L^* = (B_L - \eta C_L^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}}. \quad (30)$$

Despite the expensive signaling strategy, the H state enjoys a higher military power than the L state.

$$\tilde{D}_H(\hat{C}_H) > D_L^* \quad (31)$$

The above mentioned condition for a separating equilibrium makes sense only if L states have the resources to imitate the H states, more precisely if:

$$\eta\hat{C}_H < B_L \Leftrightarrow \frac{(B_H - B_L)}{B_L} < \frac{1}{\gamma} \left(\frac{\alpha}{\eta + \alpha} \right) \quad (32)$$

Obviously, all other things being equal, condition (32) is fulfilled if the budget gap $(B_H - B_L)$ is relatively narrow.

4.4 Signaling equilibrium and the budget limit

In the case of a large budget gap, $\frac{(B_H - B_L)}{B_L} > \frac{1}{\gamma} \left(\frac{\alpha}{\eta + \alpha} \right)$, separation occurs in a straightforward way when the H state buys $\bar{C}_H = B_L/\eta$. At this level, a L state would have used the entirety of its budget to buy howitzers, which makes any imitation impossible.

In this equilibrium the military power of the H state is:

$$\tilde{D}_H(\bar{C}_H) = (B_H - B_L) \sqrt{\frac{1 + \alpha^2}{\alpha^2}}. \quad (33)$$

The L state implements its perfect information strategy and reveals its type.

We can check that the defense level of the H state is larger than the defense level of the L state:

$$\tilde{D}_H(\bar{C}_H) > D_L^* \quad (34)$$

$$(B_H - B_L) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} > (B_L - \eta C_L^*) \sqrt{\frac{1 + \alpha^2}{\alpha^2}} \quad (35)$$

$$(B_H - B_L) > (B_L - \eta C_L^*) \quad (36)$$

which is true since the separation under resources limit required that $\frac{(B_H - B_L)}{B_L} > \frac{1}{\gamma} \frac{\alpha}{\eta + \alpha}$.

As an upshot of these, we can conclude that separation of types occurs for $C_H = \min \{ \hat{C}_H; \bar{C}_H \}$.

5 Conclusion

The war in Ukraine, which has been unfolding since February 2022, was a tremendous shock to Western policymakers, as their hopes for a peace dividend fell to pieces overnight. The war now exposes major flaws in the defense doctrines of many EU countries, which had dismissed the possibility of ever engaging in a land-based high-intensity war. The lack of readiness for such a conflict is most evident in the realm of ammunition. EU states had stocks of ammunition that would have allowed them to resist an aggression for only a few days at most. Even the US, with its tremendous military power, faced a shortage of artillery rounds and at one point had to resort to external procurement. In contrast, while there may be a shortage of some advanced air defense

systems, there is no critical shortage of main weapons such as heavy tanks, standard attack planes, and howitzers.

This paper offers a signaling explanation for the relative shortage of ammunition, spare parts, and other vital but unobserved inputs. In normal (peace) times, high-budget states might adopt a swaggering strategy of investing in fancy, high-tech, visible weapons to signal their military strength and dissuade potential aggressors from engaging in conflict. The paper analyzes this signaling game and reveals the existence of a signaling equilibrium characterized by excessive investment in main weapons and relative under-investment in ammunition by high-budget states.

In the model, we assumed *perfect complementarity* between the visible (launcher) and invisible (ammunition) inputs in the production of military power. Using a standard neoclassical production function (Hartley, 2012), involving partial complementarity and partial substitutability between visible and invisible inputs, would not change the main findings, provided that the output expansion path maintains a positive slope, which is the most plausible situation.

We have shown that in an imperfect information setting, high-budget states might sacrifice some military power to prevent low-budget states from imitating their weapon procurement strategy. This signaling cost is accompanied by an under-investment in invisible inputs, such as artillery rounds. In our analysis, the cost of signaling is related to a key parameter that captures the state's preferences for showing versus actually having military strength.

Unfortunately, what is a meaningful strategy in peacetime can become a major weakness during conflict, when actual capabilities matter much more than image. Today, EU governments and the US are obliged to ramp up the production of ammunition in an adverse industrial environment characterized by major shortages of rare-earth metals, powder, and labor. More dramatically, in 2024, Ukrainian military forces had to cope with a massive shortage of artillery rounds, resulting in major civilian and military casualties.

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